GOLIK, Mikhail Grigor'yevich, doktor sel'khoz. nauk, prof. Prinimal uchastiye KREYMERMAN, G.I., kand. tekhn. nauk; KRETOVICH, V.L., doktor biolog. nauk, prof., otv. red.; GLAZUNOV, Ye.A., red. izd-va; SHAFRANSKAYA, M.Z., red. izd-va; POLYAKOVA, T.V., tekhn. red.

[Scientific principles in the storage and processing of corn] Nauchnye osnovy khraneniia i obrabotki kukuruzy. Moskva, Izd-vo Akad. nauk SSSR, 1961. 347 p. (MIRA 14:10)
(Corn (Maize))—Storage)

KREYMERMAN, G.I.; RYUGER, A.A.

Investigation of some physicomechanical properties of artificially humidified grain. Izv. vys. ucheb. zav.; pishch. tekh. no.4:18-21 '61. (MIRA 14:8)

1. Vsesoyuznyy zaochnyy institut pishchevoy promyshlennosti, kafedra khraneniya i pererabotki zerna.
(Wheat)

KREYMERMAN, G., kand.tekhn.nauk

Conditions of shelling corn for seed and forage. Muk.-elev. prom. 27 no.7:8-10 Jl '61. (MIRA 14:7)

1. Vsesoyuznyy zaochnyy institut pishchevoy promyshlennosti.
(Corn (Maize)) (Threshing machines)

KREYMERMAN, G., kand.tekhn.nauk; INGERMAN, M., inzh.; KUZNETSOV, L.; SHONYA, M.;
NEMODRUK, I.

The DMK-1 corn threshing machine with two stages. Muk.-elev. prom. 28 no.6:6-9 Je '62. (MIRA 15:7)

1. Mirgorodskaya mashinoispytatel'naya stantsiya (for Shonya, Nemodruk). 2. Vsesoyuznyy zaochnyy institut pishchevoy promyshlennosti (for Kreymerman). 3. Vsesoyuznyy nauchno-issledovatel'skiy institut zerna i produktov yego pererabotki (for Ingerman).
(Threshing machines) (Corn (Maize))

KREYMERMAN, G.I.; RYUGER, A.A.

Investigating the free flow characteristics of the grain mass of wheat and oats in storage. Izv.vys.ucheb.zav.; pishch. tekh. no.3:21-23 '63. (MIRA 16:8)

1. Vsesoyuznyy zaochnyy institut pishchevoy promyshlennosti, kafedra tovarovedeniya, khraneniya i elevatorno-skladskogo khozyaystva.

(Wheat--Storage) (Oats--Storage)

L 17076-65 EWT(m)/EWA(d)/EWP(t)/EWP(b) Pad IJP(c)/ASD(m)-3 MJW/JD/HW

ACCESSION NR: AP4049071 S/0148/64/000/011/0158/0161

AUTHOR: Paisov, I. V.; Kreymerman, G. I.

TITLE: High-strength structural steel

B

SOURCE: IVUZ. Chernaya metallurgiya, no. 11, 1964, 158-161

TOPIC TAGS: high strength steel, low alloy steel, structural steel, superstrength steel, complex alloyed steel

ABSTRACT: To develop a heat treatment for obtaining high-strength, ductile, tough structural steels, the mechanical properties of several low-alloy steels (including complex-alloyed 45KhGSNMF, KhGS2N3M, and 45KhGS2MF steels), containing 0.4—0.45% C and various amounts of different alloying elements, have been tested after quenching from 870C to 930C and tempering at 200—600C. Test results showed that high mechanical properties in the steels investigated (tensile strength of at least 200 kg/mm^2; yield strength of at least 170 kg/mm^2; elongation of 8—10%; reduction of area of 30—35%; and notch toughness of 4—5 kgm/cm^2) can be achieved by tempering at temperatures of 250—400C, at which tempered martensite is formed.

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L 17076-65

ACCESSION NR: AP4049071

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Increasing the silicon content combined with complex alloying make it possible to raise the range of suitable tempering temperatures, while preserving the high strength of the steel. Silicon sharply raises the temperatures which bring about the irreversible temper brittleness. This allows such steels to be tempered at the 250—400C range, which is dangerous for lower alloyed steels. An excess of nickel in high-strength structural steels is undesirable since it makes it difficult to obtain maximum strength by low-temperature tempering because of the presence of a large amount of stable residual austenite. Orig. art. has: 3 figures.

ASSOCIATION: Moskovskiy institut stali i splavov (Moscow Institute of Steel and Alloys)

SUBMITTED: 13May64 ENCL: 00 SUB CODE: MM, IE

NO REF SOV: 000 OTHER: 000 ATD PRESS: 3148

Card 2/2

L 25642-65 EWT(m)/EWP(b)/EWA(d)/EWP(t) MJW/JD
ACCESSION NR: AP5003504 S/0148/65/000/001/0120/0123

21
17
B

AUTHOR: Kreymerman, G. I.; Paisov, I. V.

TITLE: Structural stability of high-strength steel

SOURCE: IVUZ. Chernaya metallurgiya, no. 1, 1965, 120-123

TOPIC TAGS: structural steel, low alloy steel, complex low alloy steel, martensite stability, mechanical property

ABSTRACT: The stability of the martensite of several high-strength steels, 45Kh, 45KhG, 45KhM, and 45KhGS, and complex 45KhGSN, 45KhGS2MF, and 45KhGS2N3M steels has been investigated. The steel specimens were oil quenched from 870—930C and tempered at 100—600C. Tempering at 150—200C brought about identical structural changes in all investigated steels. At higher temperatures (200—400C), the decomposition of martensite proceeded more slowly in complex steels, with a correspondingly slower drop in strength. Because of the greater structural and strength stability, complex low-alloy steels can be tempered in a wider temperature range than steels containing one, two, or three alloying elements, without danger of softening. For instance, 45Kh steel after tempering at 100 and 600C has a hardness H_V of 650 and 310, while 45Kh6S2MF tempered at the same temperatures has a hardness of 700 and 500. Orig. art. has: 4 figures. [MS]

Card 1/2

L 25642-65
ACCESSION NR: AP5003504

ASSOCIATION: Moskovskiy institut stali i splavov (Moscow Institute of Steel and Alloys)

SUBMITTED: 25Jun64 ENCL: 00 SUB CODE: MM, IE

NO REF SOV: 006 OTHER: 000 ATD PRESS: 3185

Card 2/2

L 14415-66 EWP(z)/EWT(m)/EWP(b)/T/EWA(d)/EWP(w)/EWI(t) MJW/JD/WB

ACC NR: AP6002126 (N) SOURCE CODE: UR/0369/65/001/006/0732/0733 58

AUTHOR: Tkachev, V. I.; Kripyakevich, R. I.; Kuslitskiy, A. B.; Kreymerman, G. I. 54

ORG: Physicomechanical Institute AN UkrSSR, L'vov (Fiziko-mekhanicheskiy institut AN UkrSSR) B

TITLE: Effect of the purity of steel and corrosion medium on low-cycle fatigue
44,14 44,55 16

SOURCE: Fiziko-khimicheskaya mekhanika materialov, v. 1, no. 6, 1965, 732-733

TOPIC TAGS: steel, corrosion, durability, hydrogen embrittlement, sulfuric acid, sodium chloride, stress concentration

ABSTRACT: The effect of the content of nonmetallic inclusions on the low-cycle fatigue of annealed ShKh15 steel produced by various processes was studied in air and in corrosive media (3% NaCl solution; 0.1 N H_2SO_4 solution; 0.1 N H_2SO_4 solution with cathodic polarization at current density $D_c = 10$ A/dm^2 corresponding to hydrogen absorption). Tests in air showed a marked divergence in the values of the durability of the purest and most contaminated steel. In the neutral medium, the durability drops by 15—25% while the effect of purity diminishes. In the acid medium, the durability drops even more (by 25—30%). Under hydrogen absorption conditions, the durability is at its minimum (about 60% of the value in air),

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L 14415-66

ACC NR: AP6002126

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and its dependence on the purity is slight; this is because the formation of brittle cracks causes a decrease in durability. As the corrosiveness of the medium increases, the influence of steel purity of low-cycle fatigue levels off, probably because additional stress concentrators which are more effective than the nonmetallic inclusions are formed. During hydrogen absorption, the inclusions act as sources of cracks. Orig. art. has: 2 figures.

SUB CODE: 11 / SUBM DATE: 17Jun65 / ORIG REF: 003

Card 2/2 10

L 2112-66 EWT(m)/EWP(w)/EWA(d)/T/EWP(t)/EWP(z)/EWP(b) IJP(c) MJW/JD

ACCESSION NR: AP5022402 UR/0369/65/000/004/0461/0464

34

AUTHOR: Kreymerman, G. I.; Paisov, I. V.

44,55 44,55

TITLE: Deformation tempering of high-strength structural steel

44,55

SOURCE: Fiziko-khimicheskaya mekhanika materialov, no. 4, 1965, 461-464

TOPIC TAGS: steel, structural steel, low alloy steel, steel treatment, thermomechanical treatment, low temperature treatment, steel property/45Kh steel, 45KhM steel, 45KhGS2N3M steel, 45KhGS2MF steel

B

ABSTRACT: The effect of additional tempering on the mechanical properties of low tempered and then cold worked alloy steels has been investigated. Specimens of 45Kh, 45KhM, 45KhGS2N3M, and 45KhGS2MF steels containing an equal amount of carbon, 0.4 to 0.45%, but different amounts of alloying elements, were austenitized, oil quenched, tempered at 200C for 1 hr, stretched at room temperature with 1% elongation, and again tempered at 200C, 300C, or 400C for 2 hr. It was found that such thermomechanical treatment increased the strength of the 45Kh and 45KhM steels by 10 dan/mm^2, and that of the complex 45KhGS2N3M and 45KhGS2MF steels by 25 dan/mm^2. The yield strength increased by as much as 90 dan/mm^2 and reached up to 240 dan/mm^2 at a satisfactory ductility. In complex alloyed steels, the strengthening effect was not an-

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L 2112-66 5

ACCESSION NR: AP5022402

nihilated by tempering at temperatures as high as 400C, which, at the same time, increased the elongation to 6—7%. Generally, the cold working of low-tempered structural steels followed by an additional tempering raises the steel yield strength to at least 220 dan/mm^2 at satisfactory ductility. The beneficial effect of cold working in alloy steels with a high content of silicon is retained with prolonged tempering at temperatures considerably higher than that of the original low tempering. Orig. art. has: 2 figures. [ND]

ASSOCIATION: Moskovskiy institut stali i splavov (Moscow Institute of Steel and Alloys) 44,55

SUB CODE: MM

ENCL: 00

SUBMITTED: 14Mar65

ATD PRESS: 4113

OTHER: 006

NO REF SOV: 004

Card 2/2

L 37941-66 EWT(m)/EWP(w)/T/EWP(t)/ETI IJP(c) JD

ACC NR: AP6023448 (N) SOURCE CODE: UR/0369/66/002/003/0336/0339

AUTHOR: Kuslitskiy, A. B.; Kreymerman, G. I.; Kokotaylo, I. V.; Starovoytov, Yu. A.; Karpenko, G. V.; Tkachev, V. I.

ORG: Physicomechanical Institute, AN UkrSSR, L'vov (Fiziko-mekhanicheskiy institut AN UkrSSR)

TITLE: Effect of metallurgical factors on the low-cycle fatigue in various media

SOURCE: Fiziko-khimicheskaya mekhanika materialov, v. 2, no. 3, 1966, 336-339

TOPIC TAGS: steel, low alloy steel, nickel containing steel, ~~vacuum degassed steel~~, ~~steel fatigue strength, steel fatigue life~~/12KhN3A steel ~~low cycle fatigue~~, ~~steel fatigue strength~~, steel (structure

ABSTRACT: Low-alloy 12KhN3A structural steel, conventionally cast or vacuum degassed, was hot-rolled into 40 mm plates or 3 mm sheets, hardened, and tempered to a tensile strength of 100 dan/mm^2, and tested for fatigue strength in the air, in a 3% NaCl aqueous solution, and in the same solution with applied cathodic polarization, the latter to promote a hydrogen absorption. A constant-amplitude, symmetrical bending at a frequency of 0.8 cps was used in the tests. The test results showed that vacuum-degassed steel had a longer fatigue life in all the investigated media than the conventionally cast steel, especially in the tests in the NaCl solution with cathodic polarization. The embrittling effect of hydrogen and, correspondingly, the difference in the fatigue life increased with increasing amplitude. Longitudinal

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L 37041-66

ACC NR: AP6023448

specimens had a longer fatigue life than that of transverse specimens. With increasing amplitude, the difference in the fatigue life of longitudinal and transverse specimens increased substantially in tests in the air, and less so in tests in NaCl solution, but noticeably decreased in the NaCl with cathodic polarization. Sheet specimens had a slightly higher fatigue life than that of plate specimens in the air and in NaCl solution, but lower in NaCl with cathodic polarization. Orig. art. has: 1 figure. [MS]

SUB CODE: 11/ SUBM DATE: 05Feb66/ ORIG REF: 002/ ATD PRESS: 5047

Card 2/2 mlp

L 45580-66 EWT(m)/EWP(w)/T/EWP(t)/ETI IJP(c) JD

ACC NR: AT6031221 (A) SOURCE CODE: UR/3107/66/000/004/0018/0022

AUTHOR: Kreymerman, G. I. (Engineer); Tomsinskiy, V. S.; (Engineer); Paisov, I. V. (Engineer)

34
29
B+1

ORG: none

TITLE: Superstrength structural steels 16

SOURCE: Nauchno-tekhnicheskoye obshchestvo mashinostroitel'noy promyshlennosti. Sektsiya metallovedeniya i termicheskoy obrabotki. Metallovedeniye i termicheskaya obrabotka, no. 4, 1966, 18-22

TOPIC TAGS: ~~superstrength~~ structural steel, silicon containing steel, steel property / 45KhNM structural steel, 45KhMF structural steel, 45KhGS structural steel, 45KhGSNMF structural steel, 45KGS2N3M structural steel, 45KGS2MF structural steel

ABSTRACT: The effect of silicon, 21 at contents of up to 2.2% on the structure and properties of alloy steels containing up to 0.8% carbon has been investigated. It was found that steel strength increases steadily with increasing silicon content. For instance, the strength of steel containing 0.2% silicon and 0.6% carbon, hardened and tempered at 400C, was 135 kg/mm^2, while that of steel containing 2.2% silicon (other conditions being the same) was 195 kg/mm^2. The effect of

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ACC NR: AT6031221

silicon is especially sharply pronounced in complex alloy steels. 45KhGSNFM, 45KhGS2N3M, and 45KhGS2MF complex alloy steels, hardened and tempered at 400C, had a tensile strength of 200—210 kg/mm^2, a yield strength of 160—185 kg/mm^2, an elongation of 8—10%, a reduction of area of 30—35%, and a notch toughness of 4—6 kgm/cm^2. Even with low tempering (200C), when the tensile strength is well over 220 kg/mm^2, the reduction of area and notch toughness remain fairly high: 27—33% and 4—6 mkg/cm^2. Irreversible temper brittleness in these steels develops at high temperature. Therefore, they can be tempered at 250—400C without danger of embrittlement. [TD] Orig. art. has: 4 figures.

SUB CODE: 11, 13/ SUBM DATE: none/ ATD PRESS: 5082

Cord 2/2 IC

L 04941-67 EWT(d)/EWT(m)/EWP(w)/EWP(t)/ETI IJP(c) EM/JD

ACC NR: AP6029688 SOURCE CODE: UR/0369/66/002/004/0464/0467

AUTHOR: Tkachev, V. I.; Kripyakevich, R. I.; Kuslitskiy, A. B.; Kreymerman, G. I.

ORG: Physics-Engineering Institute, AN UkrSSR, L'vov (Fiziko-mekhanicheskiy institut AN UkrSSR)

2U

TITLE: Effect of stress concentration on low-cycle fatigue in media

SOURCE: Fiziko-khimicheskaya mekhanika materialov, v. 2, no. 4, 1966, 464-467

TOPIC TAGS: stress concentration, material deformation, corrosive strength, hydrogenation, cyclic strength, fatigue strength

ABSTRACT: The effect of the amplitude of total deformation, ε, and of stress frequency, ν, on the low-cycle fatigue of specimens was studied with concentrators of stress, represented by 1 mm holes in the flat samples. The latter were tested in air and in corrosive and in hydrogenating environments. Concentration of stress resulted in a marked decrease of service life under low-cycle fatigue as compared with conditions of uniform stress distribution. The value N(ε), N being the number of cycles, showed the same basic dependence upon conditions as under uniform stress. The value of critical deformation decreased at a concentration of

Card 1/2

L 04941-67

ACC NR: AP6029688

stress. The dependence of the effective coefficient of stress concentration on deformation amplitude and stress frequency was determined by the ratio of the environment factors for uniform stress and concentrated stress, respectively. Orig. art. has: 3 figures.

SUB CODE: 11/ SUBM DATE: 03Mar66/ ORIG REF: 001/ OTH REF: 011

kh

Card 2/2

KREYMERMAN, I. M.

USSR (600)

Fertilizers and Manures

Correct use of fertilizer. Sakh. prom., No. 7, 1952.

9. Monthly List of Russian Accessions, Library of Congress, October 195~~1~~, Uncl.
2

KREYMERMAN, M.G.; POGORELOV, A.I.

Feed-bin device for automatic single-spindle machines. Mashinostroitel' no.10:17-18 O '59. (MIRA 13:2)
(Machine tools--Attachments)

KREYMERMAN, M.M.

103-2-3/9

AUTHOR: Kreymerman, M. M. (Moscow)

TITLE: The Determination of the Parameters of Automatic Controllers in Linear Servo-Systems, Using Given Generalized Parameters (Opredeleniye parametrov korrektiruyushchikh ustroystv lineynykh sledyashchikh sistem po zadannym znacheniyam ikh obobshchennykh parametrov)

PERIODICAL: Avtomatika i Telemekhanika, 1958, Vol. 19, Nr 2, pp.135-147 (USSR)

ABSTRACT: The theory for the selection and the determimation of the parameters of automatic controllers are given. This method makes it possible to solve the second part of the problem of a synthesis of servo-systems as formulated e.g. in Reference 1. The difference in the set-up of the problem consists of the fact that here not the characteristics of the whole system but only the optimum values of some of their most important generalized parameters which are of decisive importance for the accuracy of the servo-system (those, which influence it decisively) are given. The problem consists

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103-2-3/9

The Determination of the Parameters of Automatic Controllers in Linear Servo-Systems, Using Given Generalized Parameters

of the determination of the transmission function and of the parameters of the automatic controller if the following is given: the course of the logarithmic amplitude characteristics (l.A.C.), of the type considered, the transmission function of the known part of the structural computing circuit of the servo-system (selected from technical considerations) and the numerical values of ω_c, φ_c and D_c. (ω_c - gain cross-over, φ_c - stability reserve in the phase, D_c - relative figure of merit (quality)). A table is given which shows the basic formulae of computation. These formulae can also be used for servo-systems with an astatic operation of second order and can be extended to servo-systems with other transmission functions of the open circuits. At the end examples for the computation of the parameters of controllers connected in series and in parallel and of their combinations are given. There are 10 figures, 2 tables, and 2 references, 2 of which are Slavic.

Card 2/2

SUBMITTED: December 26, 1956

1. Automatic controllers-Mathematical analysis

KREYN, G.F.

"Use of electric current for direct action on the fuel bed in underground coal gasification without mining" by S.T. Bondarenko and others. Reviewed by G.F.Krein. Podzem.gaz. ugl. no.3:76 '59. (MIRA 12:12)

(Coal gasification, Underground)

(Bondarenko, S.T.)

KREYN, G.F.

Intensity of blast flow toward the gas evacuation boreholes during the process of countercurrent filtrating connection linking. Trudy VNIIPodzemgaza no.13:66-74 '65.

(MIRA 18:8)

1. Laboratoriya tekhnologii podzemnoy gazifikatsii ugley Vsesoyuznogo nauchno-issledovatel'skogo instituta podzemnoy gazifikatsii ugley.

REZNIKOV, A.D.; BRUSHTEYN, N.Z., kand.tekhn.nauk; MIRINGOF, N.S., kand.tekhn.nauk; KREYN, G.F.

Experience in conducting the combined connection linking at the "Podzemgaz" Plant in Shatsk. Nauch. trudy VNII Podzemgaza no.6:86-95 '62. (MIRA 15:11)

1. Laboratoriya elektrotermicheskaya i laboratoriya gazifikatsii burykh ugley Vsesoyuznogo nauchno-issledovatel'skogo instituta podzemnoy gazifikatsii ugley.
(Lvov-Volyn' Basin—Coal gasification, Underground)

KREYN, G.F.

Study of various systems and parameters for electrical linking on an electrolytic model. Nauch. trudy VNIIPodzemgaza no.9:85-91 '63. (MIRA 16:11)

1. Laboratoriya teplotekhniki i energetiki Vsesoyuznogo nauchno-issledovatel'skogo instituta podzemnoy gazifikatsii ugley.

KREYN, M. G.

O nekotorykh voprosakh teorii momentov.

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O rynde teylora, opredelyayushchem analiticheskuyu funktsiyu, regulyarnuyu v oblasti, ogranichennoy neskol'kimi kruzhkami. Kazan', Izv. fiz. -matem. O-VA (3), 2 (1927). 50-57.

Ueber eine neue klasse von hermiteschen formen und euber eine verallgemeinerung des trigonometrischen momentenproblems, I. IAN, ser fiz. -matem. (1933), 1259-1275 .

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Sur les operateurs differentiels autoadjoints et leurs fonctions de Green symetriques. Matem. SB., 2 (44), (1937), 1023-1072.

K teorii nailuchshego priblizheniya periodicheskikh funktsiy. DAN, 18 (1938). 245-250.

O nailuchshey approksimatsii nepreryvnykh differentsiruemykh funktsiy na vsey veshchestvennoy osi. DAN, 18 (1938), 619-624.

O predstavlenii funktsiy integralami fury-stilt'esa. Kuybyshev, Uchen. zap. Ped. i uchitel'sk. in-ta, 7 (1943), 123-148.

KREYN, M. G. con't.

Ob obobshchennoy probleme momentov. DAN, 44 (1944), 239-243.

Ob odnom obobshchenii issled-ovaniy G. Szego, V. I. Smirnova i A. N. Kolmogorova. DAN, 46 (1945), 95-98.

K teorii tselykh funktsiy ekspone-ntsial'nogo tipa. IAN, ser. matem., 11 (1947), 309-326.

O funktsiyakh grina, polozhitel'nykh v smysle mersera. DAN, 1 (1936), 55-58.

Ob ostsillyatsionnykh differentsial'nykh operatorakh. DAN, 4 (1936), 379-382.

Sur les developpements des fonctions arbitraires en series de fonctions fondamentales dun probleme aux limites quelconque. Mate. SB., 2 (44), (1937), 923-934.

Sur les operateurs differentiels, auto-adjoints et leurs fonctions de green symetriques. Matem. SB., 2 (44), (1937), 1023-1072.

O nesimmetri cheskikh ostsillyatsionnykh funktsiyakh grina obyknovennykh differentsial'nykh operatorov. Dan, 25 (1939), 643-646.

O stsillyatsionnyye teoremy dlya obyknovennykh lineynykh differentsial'nykh operatorov prozvol'nogo poryadka, DAN, 25 (1939). 717-720.

KREYN, M. G., Con't.

O defektnykh chislakh zamknutykh operatorov. DAN, 56 (1947), 559 - 652.

Sur les equations integrales chargees. C. R. Acad. Sci., 201 (1935), 24-26.

Sur quelques applications des noyaux de kellog aux problemes d'oscillation. Khrk, zap. matem. T-va (4), 11 (1935), 3-20.

Pro pozitivni additivni funktsionali v liniynikh normovanikh prostorakh. Khrk., zap. matem. t-va (4), 14 (1937), 227-237.

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KREYN, M. G., con't.

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O polozhitel'nykh funktsionalakh na pochti periodicheskikh funktsiyakh. DAN, 30 (1941), 9-12.

Ob odnom novom svoystve operatora shturma-liuvillya. Odessa, trudy un-ta, 3 (1941), 15-22.

O predstavlenii funktsiy integralami fury=stilt'esa. Kuybyshev, Uchen. zap. ped. in-ta, 7 (1943).

Ob ermitovykh operatorakh s defekt=indeksami, ravnymi e dinitse, II. DAN, 44 (1944), 143-146.

Ob odnom zamechatel'nom klasse ermitovykh operatorov DAN, 44 (1944), 191-195.

Ob obobshchennoy probleme momentor. DAN, 44 (1944), 239-243.

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KREYN, M. G., con't.

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Moscow-Leningrad, 1948

KREYN, M. G.

Krein, M. Sur un théorème de M. Vygodsky. Rec. Math. [Mat. Sbornik] N.S. 18(60), 447–450 (1946). (Russian. French summary)

The theorem [same Rec. N.S. 16(58), 73–80 (1945); these Rev. 7, 75] states that a continuous closed spherical curve L is the tangent indicatrix of a closed curve having a tangent at each point, if and only if L is contained in no hemisphere. Vygodsky gave a geometric proof; the author gives an analytic proof which also holds in n dimensions, $n \geq 3$. *R. P. Boas, Jr.* (Providence, R. I.).

[illegible] Mathematical Reviews, Vol [illegible]

KREYN, M

Krein, M. Concerning the resolvents of an Hermitian operator with the deficiency-index (m, m). C. R. (Doklady) Acad. Sci. URSS (N.S.) 52, 651-654 (1946).

The author outlines a method of obtaining the forms of the generalized resolvents R_z of a Hermitian operator A on a Hilbert space $\mathfrak{H}$ with deficiency index (m, m), generalizing the result for deficiency index $(1, 1)$ obtained in a previous paper [same C. R. (N.S.) 43, 323-326 (1944); these Rev. 6, 131]. Generalized resolvents R_z correspond to spectral functions $E(t)$ of A by the formula $R_z\varphi = \int_{-\infty}^{\infty}(t-z)^{-1}dE(t)\varphi$ for $\mathfrak{J}z>0$. The set of all generalized resolvents of A is given by

$$R_z = R_z^0 + \sum_{i,k=1}^{m} [\cdot, \varphi_i(\bar{z})]h_{ik}(z)\varphi_k(z), \qquad \mathfrak{J}z>0,$$

where R_z^0 is the resolvent of a certain self-adjoint extension A^0 of A, $\varphi_i(z)$ are m linearly independent solutions of $A^*\varphi - z\varphi = 0$ (in $\mathfrak{H}$ for each nonreal z), where A^* is the operator maximally adjoined to A; $h_{ik}(z)$ forms a matrix of complex-valued functions of z, the reciprocal of the matrix of $q_{ik}(z)+f_{ik}(z)$, with

$$q_{ik}(z) = [(z-z_0)\varphi_i(z)+iy_0\varphi_i(\bar{z}_0), \varphi_k(z_0)],$$

z_0 is an arbitrary regular point of R_z^0, and $f_{ik}(z)$ are holomorphic and such that $\mathfrak{J}(\sum_{ik}\xi_i f_{ik}(z)\bar{\xi}_k) \geqq 0$ for $\mathfrak{J}z>0$. The case in which the operator A is positive, i.e., $[A\varphi, \varphi] \geqq 0$ for φ in $\mathfrak{D}(A)$, is also discussed.

T. H. Hildebrandt.

Source: Mathematical Reviews, Vol 8, No. 5

KREYN, M.

Krein, M. On a general method of decomposing Hermite-positive nuclei into elementary products. C. R. (Doklady) Acad. Sci. URSS (N.S.) 53, 3–6 (1946).

It is stated without proof that the following result can be proved using methods previously established by the author [same C. R. (N.S.) 44, 219–222 (1944); these Rev. 6, 270]. If there exists in Hilbert space $\mathfrak{H}$ (not necessarily complete) a Hermitian operator A and linear, not necessarily continuous, functionals $\Phi_j(f;\lambda)$, $j=1,\cdots,p$; $\lambda\epsilon(-\infty,\infty)$, which depend analytically on λ for every $f\epsilon\mathfrak{H}$, and if $Ag-\lambda_0 g=f_0$ has a solution g_0 in the domain $\mathfrak{D}(A)$ of A if and only if $\Phi_j(f_0,\lambda_0)=0$, $j=1,\cdots,p$, $\lambda_0\epsilon(-\infty,\infty)$, then there exists a matrix valued function $\mathfrak{T}(\lambda)=\|\tau_{jk}(\lambda)\|$, $j,k=1,\cdots,p$, such that

$$(1)\qquad (g,f)=\sum_{j,k=1}^{p}\int_{-\infty}^{\infty}\Phi_j(g;\lambda)\overline{\Phi_k(f;\lambda)}\,\tau_{jk}(\lambda)d\lambda$$

for every pair $g,f\epsilon\mathfrak{H}$, and (2) $\sum_{j,k=1}^{p}\tau_{jk}(\lambda)\xi_j\bar{\xi}_k$ is a nondecreasing function of λ for arbitrary complex $\xi_1,\cdots,\xi_p$. If $(Af,f)\geq 0$ for $f\epsilon\mathfrak{D}(A)$, there exists $\mathfrak{T}(\lambda)$ satisfying (1) and (2) such that $\tau_{jk}(\lambda)=0$, $j,k=1,\cdots,p$, $\lambda\epsilon(-\infty,0)$. As an application it is indicated how results of the following type can be obtained. A necessary and sufficient condition that $F(x)$, continuous in $(0,\infty)$, have the form $\int_{-\infty}^{\infty}\cos\lambda x\,d\tau(\lambda)$, where $\tau(\lambda)$ is a certain nondecreasing function of λ, is that the nucleus (kernel) $F(x+y)+F(|x-y|)$ be positive definite.

R. A. Leibler (Princeton, N. J.).

Source: Mathematical Reviews, Vol 8, No. 5

KREYN, M. G.

Source: Mathematical Reviews, Vol 10 No. 3

Krein, M. G., and **Krasnosel'skii, M. A.** Fundamental theorems on the extension of Hermitian operators and certain of their applications to the theory of orthogonal polynomials and the problem of moments. Uspehi Matem. Nauk (N.S.) 2, no. 3(19), 60–106 (1947). (Russian)

Some useful notions are introduced. By the "inclination" [raztvor] of two linear varieties $\mathfrak{L}_1$, $\mathfrak{L}_2$ in Hilbert space $\mathfrak{H}$ is understood the number

$$\theta(\mathfrak{L}_1, \mathfrak{L}_2) = \max\left[\sup_{f\in\mathfrak{L}_1, |f|=1} \rho(f, \mathfrak{L}_2), \sup_{f\in\mathfrak{L}_2, |f|=1} \rho(f, \mathfrak{L}_1)\right],$$

where $\rho(f, \mathfrak{L}_i) = \inf |f-g|$ for $g\in\mathfrak{L}_i$, $i=1, 2$. We have $\theta(\mathfrak{L}_1, \mathfrak{L}_2) = \theta(\mathfrak{N}_1, \mathfrak{N}_2)$, where $\mathfrak{N}_i$ is the closed subspace orthogonally complementary to $\mathfrak{L}_i$. If $\theta(\mathfrak{L}_1, \mathfrak{L}_2) < 1$, then $\mathfrak{N}_1$, $\mathfrak{N}_2$ have equal dimensions. For a linear transformation A with domain $\mathfrak{D}(A) \subset \mathfrak{H}$ the complex number λ is said to be of regular type if $|(A-\lambda I)f| \geq k_\lambda |f|$ for every $f\in\mathfrak{D}(A)$ and some $k_\lambda > 0$ [see Krasnosel'skii, Doklady Akad. Nauk SSSR (N.S.) 56, 559–561 (1947); these Rev. 9, 242]. The set of all λ of regular type is open. If G is a connected open set composed of numbers of regular type, the closed orthogonal complement to $\mathfrak{R}(A-\lambda I)$ has the same dimension e for all $\lambda\in G$. The cardinal number e is called the deficiency index of A in G.

The ideas of M. Riesz [Acta Litt. Sci. Szeged 1, 209–225 (1923)] are generalized to the moment problem $\int_M z^n d\sigma(z) = \mu_n$, where M is a closed set of complex numbers and $\sigma(E)$ a countably additive nonnegative function of Borel subsets of M with $\int_M |z|^n d\sigma(z) < \infty$, and these ideas are translated into the language of the Hilbert space $\mathfrak{L}_\sigma$ composed of all $f(z)$ with $\int_M |f(z)|^2 d\sigma < \infty$, $\int_M |f(z)| d\sigma < \infty$ and with the scalar product $(f, g) = \int_M f\bar{g} d\sigma$. Let $\mathfrak{H}_\sigma$ be the closure, in $\mathfrak{L}_\sigma$, of the set $\mathfrak{P}$ of all polynomials $P(z)$. The orthonormalized sequence $D_0(z) = 1$, $D_1(z)$, $D_2(z)$, $\cdots$ of $1, z, z^2, \cdots$, and the transformation $AP(z) = zP(z)$ are considered. The deficiency

Source: Mathematical Reviews, CARD 2/2 Vol 10 No, 3

KREIN, M. G.

indices of A are 0 or 1. The sum $\sum_{k=0}^{\infty}|D_k(\lambda)|^2$ converges or diverges at the same time at all points λ of a component W of the complement of M. Relations between the deficiency index of A in W and of $(z-\lambda)^{-1}$ belonging to $\mathfrak{H}$, are studied. If the equality $\int_M(z-\lambda)^{-1}d\omega(z)=0$ for all $\lambda \bar{\in} M$ implies $\omega(z)=0$ for all complex countably additive $\omega(E)$ defined on Borel subsets of M, the set M is said to satisfy the (α)-condition. This case is studied and applied to the classical Hamburger moment problem: $s_k=\int_{-\infty}^{\infty}t^k d\sigma(t)$, $k=0, 1, 2, \cdots$. A new proof of the known existence theorem ($\sum_{i,k=0}^{n}s_{i+k}\xi_i\bar{\xi}_k$ nonnegative definite for all n) is given by defining a scalar product for polynomials $P(t)=\sum_k\xi_k t^k$, $Q(t)=\sum_k\eta_k t^k$ by the Hankel-bilinear form $(P,Q)=\sum_{i,k}s_{i+k}\xi_i\bar{\eta}_k$ [see J. A. Shohat and J. D. Tamarkin, The Problem of Moments, Math. Surveys no. 2, New York, 1943, p. 1; these Rev. 5, 5]. This leads to some Hilbert subspaces of $\mathfrak{H}$. The transformation A is Hermitian. The spectral representation of the resolvent of a self-adjoint extension of A (obtained, if necessary, by extending $\mathfrak{H}$ [see Neumark, Bull. Acad. Sci. URSS. Sér. Math. [Izvestia Akad. Nauk SSSR] 4, 53–104 (1940); these Rev. 2, 104] immediately furnishes the solution of the moment problem. A criterion, in terms of A, for the existence of a solution is given. There are also new proofs of the classical approximation theorem of M. Riesz [loc. cit.], and of a part of the Nevanlinna theorem. Some results on $(\alpha A+\beta I)(\gamma A+\delta I)^{-1}$ which generalize the topic studied previously by Krein [Rec. Math. [Mat. Sbornik] N.S. 20(62), 431–495 (1947); Mat. Sbornik N.S. 21(63), 365–404 (1947); these Rev. 9, 515] are used as a tool. Krein's theory of extension of generalized Hermitian operators [cf. loc. cit.] is developed again. Applications to the moment problem $\int_{-1}^{1}t^k dF(t)=s_k$, to the classical Stieltjes problem, to the trigonometric moment problem and to a generalized one are given.

O. M. Nikodým.

~~KREYN, M.~~

KREYN, M. G.

Krein, M. A contribution to the theory of entire functions of exponential type. Bull. Acad. Sci. URSS. Sér. Math. [Izvestia Akad. Nauk SSSR] 11, 309–326 (1947). (Russian. English summary)

A function $f(z)$ is said to be of class N in a region if $\log^+|f(z)|$ has a harmonic majorant there. The author's principal results concern entire functions belonging to class N in half planes. In the first place he shows that, if $f(z)\in N$ in the upper half plane, there is a real number μ such that $\limsup_{r\to\infty} r^{-1}\log|f(re^{i\varphi})| = \mu\sin\varphi$, $0<\varphi<\pi$, uniformly in any closed interior angle, while in any angle in which $f(z)$ has no zeros, $\lim r^{-1}\log|f(re^{i\varphi})| = \mu\sin\varphi$. He next shows that a necessary and sufficient condition for an entire function $f(z)$ to belong to N in both the upper and lower half planes is that $f(z)$ is of exponential type and satisfies $\int_{-\infty}^{\infty}(1+x^2)^{-1}\log^+|f(x)|\,dx<\infty$.

Special subclasses of N are (1) the class of $f(z)$ with real simple zeros α_k, such that $1/f(z) = P(z) + z^s\sum_1^\infty c_k/(z-\alpha_k)$, with $\sum|c_k|<\infty$, $P(z)$ a polynomial; (2) the class of $f(z)$ with simple zeros α_k such that $\sum\Im(\alpha_k)/|\alpha_k|^2<\infty$ and $1/f(z)=\sum_{k=1}^\infty c_k/(z-\alpha_k)$, $\sum|c_k|/|\alpha_k|<\infty$.

The results are obtained by mapping a half plane on a circle, so that known results on functions of class N in a circle can be applied [Nevanlinna, Eindeutige analytische Funktionen, Springer, Berlin, 1936, chap. 7. Smirnov, J. Soc. Phys.-Math. Léningrade [Žurnal Leningradskogo Fiz.-Mat. Obščestva] 2, no. 2, 22–37 (1928)]. An auxiliary theorem proved in the paper is that, if $F(z)\in N$ in $|z|<1$, and Γ is a path going to $z=1$ inside $|z|<1$ and making a positive angle θ with the real axis at $z=1$, then $\limsup_{z\to1}|1-z|\log|F(z)| = d\cos\theta$, where d does not depend on Γ.

R. P. Boas, Jr. (Providence, R. I.).

Source: Mathematical Reviews, 1948, Vol 9, No. 4

KREYN, M. G.

Krein, M. The theory of self-adjoint extensions of semi-bounded Hermitian transformations and its applications. I. Rec. Math. [Mat. Sbornik] N.S. 20(62), 431–495 (1947). (Russian. English summary)

The author investigates the set of all self-adjoint extensions $\tilde{T}$ of a semi-bounded (from below) closed Hermitian transformation T, with $\overline{\mathfrak{D}T}=\mathfrak{H}$, $m(T)=\inf (Tf,f)/(f,f)>-\infty$ for $f\epsilon\mathfrak{D}T$, $f\neq 0$ in Hilbert space $\mathfrak{H}$. Their existence (with the same m) was proved by M. H. Stone [Linear Transformations in Hilbert Space, Amer. Math. Soc. Colloquium Publ., v. 15, New York, 1932, theorem 9.21] and by K. Friedrichs [Math. Ann. 109, 465–487, 685–713 (1934)] and H. Freudenthal [Nederl. Akad. Wetensch., Proc. 39, 832–833 (1936)]. A new proof is given. The problem is reduced to a positive transformation S, with $\overline{\mathfrak{D}S}=\mathfrak{H}$, and this to generalized bounded closed Hermitian A, with $\mathfrak{D}A=\overline{\mathfrak{D}A}$, by putting $A=(I-S)(I+S)^{-1}$. The transformation A is called Hermitian if $(Af,g)=(f,Ag)$ for every f,g in the domain $\mathfrak{D}A$ of A, even if $\mathfrak{D}A\neq\mathfrak{H}$; this is equivalent to the relation $A\wedge A$, as defined by F. J. Murray [Trans. Amer. Math. Soc. 37, 301–338 (1935)], where A is a transformation "from $\mathfrak{H}$ into $\mathfrak{H}$." If H, H_1 are bounded self-adjoint ($\mathfrak{D}H=\mathfrak{D}H_1=\mathfrak{H}$), $H\leqq H_1$ means $(Hf,f)\leqq(H_1f,f)$ for $f\epsilon\mathfrak{H}$. If $\mathfrak{N}$ is a closed subspace of $\mathfrak{H}$ and $H\neq 0$, as above, with $H\geqq 0$, $H_{\mathfrak{N}}$ denotes $H^{\frac{1}{2}}P_{\mathfrak{L}}H^{\frac{1}{2}}$, where $\mathfrak{L}$ is the subspace composed of all the vectors f for which $H^{\frac{1}{2}}f\epsilon\mathfrak{N}$ (here $H^{\frac{1}{2}}\geqq 0$ and $P_{\mathfrak{L}}$ is the projection on $\mathfrak{L}$).

The fundamental theorem is as follows. For every A with norm $\|A\|=1$ there exist two self-adjoint extensions A_μ, A_M with norm 1 and with $\mathfrak{D}A_\mu=\mathfrak{D}A_M=\mathfrak{H}$, such that a bounded self-adjoint $\tilde{A}$ with $\mathfrak{D}\tilde{A}=\mathfrak{H}$ is an extension of A with $\|\tilde{A}\|=1$ if and only if $A_\mu\leqq\tilde{A}\leqq A_M$; we have $A_\mu=\tilde{A}-(I+\tilde{A})_{\mathfrak{N}'}$, $A_M=\tilde{A}+(I-\tilde{A})_{\mathfrak{N}'}$, where $\mathfrak{N}'=\mathfrak{H}\ominus\mathfrak{D}A$.

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Source: Mathematical Reviews, 1/2 Vol 7 No. 7

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The proof is based on the following lemma. For H and $\mathfrak{N}$ as above, the set of all bounded self-adjoint C, with $\mathfrak{D}C=\mathfrak{H}$, $C\leqq H$ and with the range $\mathfrak{R}C\subset\mathfrak{N}$, contains the transformation $H_{\mathfrak{N}}$ which is "maximal" in the sense $C\leqq H_{\mathfrak{N}}$. The $H_{\mathfrak{N}}$ is studied explicitly. In the proofs, throughout the paper, frequent use is made of the identity $(Hf, f)=\|H^{\frac{1}{2}}f\|^2$ and of generalized orthometrics introduced by Friedrichs [loc. cit.]. Define $\mathfrak{D}[T]$ as the set of all $f\in\mathfrak{H}$ for which there exists a sequence $\{f_n\}$ in $\mathfrak{D}T$ with $f_n\rightarrow f$ and $(T(f_n-f_m), f_n-f_m)\rightarrow 0$ for $n, m\rightarrow\infty$. Then for $f, g\in\mathfrak{D}[T]$, $T[f, g]=\lim (Tf_n, g_n)$ is independent of the choice of $\{f_n\}$ and $\{g_n\}$ [Friedrichs, loc. cit.] and $\mathfrak{D}[T]$ is a complete Hilbert space with the orthometric $(g, f)_T=(g, f)+T[g, f]$. For S, if we put $S_\mu=(I-A_\mu)(I+A_\mu)^{-1}$, and analogously for S_M, then for every positive self-adjoint extension $\tilde{S}$ of S and every $a>0$ we have $(S_\mu+aI)^{-1}\leqq(\tilde{S}+aI)^{-1}\leqq(S_M+aI)^{-1}$, $\mathfrak{D}[S_\mu]\subset\mathfrak{D}[\tilde{S}]\subset\mathfrak{D}[S_M]$. A characteristic property is given for S_μ, and, in the case $m(S)>0$, also for S_M. This case is studied explicitly and there are theorems like $\mathfrak{D}S_M=\mathfrak{D}S+\mathfrak{N}_0$, $\mathfrak{D}[S_M]=\mathfrak{D}[S]+\mathfrak{N}_0$, $\mathfrak{D}S^*=\mathfrak{D}S_\mu+\mathfrak{N}_0$, where $\mathfrak{N}_0$ is the set of all roots φ of $S^*\varphi=0$.

The case of a *half-bounded* transformation with finite deficiency index is studied, ~~and~~ also Hermitian H with "gaps." If H is Hermitian, $\overline{\mathfrak{D}H}=\mathfrak{H}$, then (a, b) is called a gap if $\|Hf-\frac{1}{2}(b+a)f\|\geqq\frac{1}{2}(b-a)\|f\|$ for $f\in\mathfrak{D}H$. J. W. Calkin's results [Duke Math. J. 7, 504–508 (1940); these Rev. 2, 224] are completed. If $\lambda_1<\lambda_2<\cdots<\lambda_s$ are regular points (λ "regular" means that there exists at least one self-adjoint extension whose resolvent set contains λ), and $p_1, \cdots, p_s$ are natural numbers with $p_1+\cdots+p_s=e(H)$ (where (e, e) is the deficiency index of H), then there exists at least one self-adjoint extension of H for which λ_j $(j=1, \cdots, s)$ are characteristic values with multiplicity at least p_j. Transformations with discrete spectrum are studied. An important remark [top of p. 437] is incorrect, and, therefore, the proofs of several integral, inf. and sup. theorems by the author do not appear sufficiently clear to the reviewer.

O. M. Nikodým (Gambier, Ohio).

Source: Mathematical Reviews, 3/2 Vol 8 No. 9

KREYN, M. G.

Krein, M. G. The theory of self-adjoint extensions of semi-bounded Hermitian transformations and its applications. II. Mat. Sbornik N.S. 21(63), 365–404 (1947). (Russian)

[Cf. the preceding review.] Self-adjoint boundary problems in the differential form

$$(1)\quad f^{[2n]}=p_nf-\frac{d}{dx}\left\{p_{n-1}f'-\frac{d}{dx}\left[p_{n-2}f''-\cdots-\frac{d}{dx}\left[p_1f^{(n-1)}-\frac{d}{dx}(p_0f^{(n)})\right]\cdots\right]\right\}$$

are investigated, in the direction followed by M. H. Stone [Linear Transformations in Hilbert Space, Amer. Math. Soc. Colloquium Publ., v. 15, New York, 1932], where $p_k(x)$ are real-valued measurable functions in (a, b) such that $\int_a^b dx/|p_0(x)|<\infty$, $\int_a^b|p_k(x)|dx<\infty$ $(k=1, \cdots, n)$. If p_i are sufficiently regular, (1) becomes

$$\sum_{k=0}^{n}(-1)^k\frac{d^k}{dx^k}\left(p_{n-k}\frac{d^kf}{dx^k}\right),$$

to which form the general linear Lagrange-self-adjoint differential classical expression of even order can be reduced [Jacobi, Bertrand]. To manage (1) by Hilbert-space methods put $f^{[0]}=f$, $f^{[k]}=df^{[k-1]}/dx$ $(k=0, \cdots, n-1)$, $f^{[n]}=p_0\cdot df^{[n-1]}/dx$, $f^{[n+k]}=-df^{[n+k-1]}/dx+p_kf^{[n-k]}$ $(k=1, \cdots, n)$ and define D^* as the set of all complex $f\varepsilon L_2(a, b)$ for which $f^{[0]}, \cdots, f^{[2n-1]}$ are absolutely continuous in $a\leqq x\leqq b$ and $f^{[2n]}\varepsilon L_2(a, b)$. For any $h\varepsilon L_2(a, b)$ and any $0\leqq x_0\leqq 1$ the system $f^{[2n]}=h(x)$, $f^{[k]}(x_0)=C_k$ $(k=0, \cdots, 2n-1)$, where C_k are given complex constants, admits one and only one solution $f\varepsilon D^*$. Define the operator T in the Hilbert space $L_2(a, b)$, on the set D of all $f\varepsilon D^*$ with $f^{[k]}(a)=f^{[k]}(b)=0$ $(k=0, \cdots, 2n-1)$, by putting $Tf=f^{[2n]}$. It is Hermitian with deficiency index $(2n, 2n)$, its domain D is everywhere dense in L_2, and the domain of its adjoint T^* is D^*. Self-

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Source: Mathematical Reviews, 1/3 Vol 7 No. 7

adjoint extensions $\tilde{T}$ of T are studied, in close relation with J. W. Calkin's paper [Trans. Amer. Math. Soc. 45, 369–442 (1939)], by attaching the vector $\xi=(\xi_1, \cdots, \xi_{4n})$ in the $4n$-dimensional unitary space $\mathfrak{E}$ to the boundary condition $\xi_k=f^{(k-1)}(a)$, $\xi_{2n+k}=f^{(k-1)}(b)$ $(k=1, \cdots, 2n)$, for $f\varepsilon D^*$. It is shown that there is a one-to-one correspondence between (1') self-adjoint homogeneous boundary conditions, (2') self-adjoint extensions $\tilde{T}$ and (3') subspaces $\Pi(\tilde{T})$ of $\mathfrak{E}$ such that Π is the orthogonal complement of $U(\Pi)$, where U is the unitary transformation in $\mathfrak{E}$ carrying the vector $(\xi_1, \cdots, \xi_{4n})$ into $(\eta_1, \cdots, \eta_{4n})$; here $\eta_k=\xi_{2n+1-k}$, $\eta_{n+k}=-\xi_{n-k+1}$, $\eta_{2n+k}=-\xi_{4n-k+1}$, $\eta_{3n+k}=\xi_{3n-k+1}$ $(k=1, \cdots, n)$. If (2) $p_n(x)\geqq 0$ in $a\leqq x\leqq b$, then T is half-bounded from below [see the review of part I]. It is stated without proof that the converse also holds true. In case (2) every $\tilde{T}$ has a discrete spectrum. In the case where 0 does not belong to the spectrum the corresponding Green's function $G(x, \xi)$ is built up by means of the Cauchy-function for (1) (i.e., $\Phi(x, \xi)$ which, considered as a function of x, is the solution of $y^{(2n)}=0$ with $\Phi^{(k)}(x, \xi)|_{x=\xi}=0$ $(k=0, \cdots, 2n-2)$, and $\Phi^{(2n-1)}(x, \xi)|_{x=\xi}=1$).

Source: Mathematical Reviews, Vol. 9 No. 7

KREYN, M.G.

Krein, M. G., and Rutman, M. A. Linear operators leaving invariant a cone in a Banach space. Uspehi Matem. Nauk (N.S.) 3, no. 1(23), 3–95 (1948). (Russian)

This paper gives proofs and extensions of earlier work of the authors on the existence and properties of characteristic vectors and characteristic numbers of a monotone linear operator A in a partially ordered Banach space B. The various hypotheses of the theorems describe the nature of the cone K of nonnegative elements and impose conditions, such as complete continuity, on the operator A. The results, in particular § 6, extend such classical theorems as those of Frobenius and Perron on matrices with nonnegative elements, and the theorem of Jentzsch on integral operators with positive kernel. Section 9 has some results on certain nonlinear, completely continuous operators. The existence theorems are derived from the fixed point theorem for convex compact subsets of a locally convex linear topological space.

Section 1 deals with semi-groups in a Banach space B; that is, with convex sets K which are semi-groups under addition in B. The notation $x \leqq y$ means that $y-x \varepsilon K$; $x \ll y$ means that $y-x \varepsilon$(interior of K). Define K^* as the set of f such that $f \varepsilon B^*$ and $f(x) \geqq 0$ for all x in K. Call f in K^* positive (strictly positive) if there exists x in K (if for all $x \neq 0$ in K) $f(x) > 0$. Typical preliminary results are as follows. Theorem 1.1. Let K be a cone, not all of B, and let G be a linear subspace of B containing an interior point of K; then every positive linear functional on G can be extended to a positive linear functional on B. Theorem 1.2. If K_1 is a semi-group with interior and K_2 a semi-group not containing an interior point of K_1, then there exists a positive f in K_1^* with $-f$ in K_2^*.

Section 2 introduces cones; that is, closed semi-groups K such that x and $-x$ both in K implies $x=0$. Call K normal if there exists $\delta > 0$ such that $x_i \varepsilon K$ and $\|x_i\| = 1$, $i = 1, 2$, imply $\|x_1 + x_2\| > \delta$. Call K minihedral if $\sup(x, y)$ and $\inf(x, y)$ exist for every x, y in K. If K has an interior, K^* is normal. If, moreover, K is minihedral, K^* is minihedral. If K has an interior point u, K is normal if and only if the interval $\{x \mid -u \leqq x \leqq u\}$ is contained in a sphere. In

Source: Mathematical Reviews, Vol 10 No. 4

a finite-dimensional space a cone K is minihedral if and only if there exist linearly independent elements for which K is the set of nonnegative linear combinations of these elements. Kakutani's theorem on the representation of abstract M spaces can be rephrased as follows: a Banach space B is isomorphic to the space of continuous functions on a compact Hausdorff space if and only if there exists in B a normal, minihedral cone with interior.

Section 3 studies common fixed points and characteristic vectors of an Abelian family of operators and of their adjoints. (In sections 1–8 all operators are additive and continuous.) Theorem 3.3. Let K be a semi-group with interior and let G be a commuting family of linear transformations A with $AK\subseteq K$. Then there exists a positive φ in K^* which is a common characteristic vector of all the conjugate operators A^*, A in G; in fact, for each A in G there exists for each A a positive λ_A such that $A^*\varphi=\lambda_A\varphi$. Theorem 3.1. If in addition there is a common fixed point $u\gg0$ of all A in G, then φ is a common fixed point of the A^*, A in G. This has as a corollary the existence of Banach limits for bounded real functions on Abelian groups.

Section 4 introduces the complex extension $\bar{B}$ of a given real Banach space B: $\bar{B}$ is the space of couples $z=x+iy$ of elements of B with the usual norm

$$|z|=\max_{0\leq\mu\leq 2\pi}|x\cos\mu+y\sin\mu|;$$

and the extension $\bar{A}$ of A: $\bar{A}z=Ax+iAy$. The spectrum, resolvent set, and resolvent of A are defined to be the corresponding quantities for $\bar{A}$. Call K a reflexive cone if B is a reflexive space (so $K=K^{**}$) and both K and K^* have interiors. Theorem 4.1. Let K be a reflexive cone and A an operator mapping the interior of K into itself and such that $x\epsilon K$ and $Ax=0$ implies $x=0$. Then there exist $x\neq0$ in K, $f\neq0$ in K^*, and a positive number ρ such that $Ax=\rho x$, $A^*f=\rho f$, and the spectrum S_A of A lies in the circle $|\lambda|\leq\rho$.

These results are applied in §5 to Lorentz mappings in Hilbert space H. Fix an orthogonal basis $\{e_i\}$ in H; for $x=\sum_{i=1}^{\infty}\xi_ie_i$, let $Jx=\xi_1e_1-\sum_2^{\infty}\xi_ie_i$. Then K, the set of x such that $(Jx,x)\geqq0$, $(e_1,x)\geqq0$, is a reflexive cone; in fact, $K^*=K$. If $(Ju,u)>0$, then $[x,y]=-(Jx,y)$ is an inner product defining the original topology in the hyperplane $H_u=$(set of all y such that $(Ju,y)=0$). Call Γ a Lorentz mapping if it is a one-to-one function carrying H onto itself so that $(J\Gamma x,\Gamma y)=(Jx,y)$ for each x, y in H. It is shown that every Lorentz mapping is linear and either Γ or $-\Gamma$ maps K into itself. Theorem 5.1. Let Γ be a Lorentz mapping taking K into K; then Γ has a characteristic vector v

Source: Mathematical Reviews, Vol 10 No. 4

Krein, M. G. & Ru[illegible], M. A.

in K and a characteristic number $\rho \geqq 1$ for which precisely one of the following two cases holds: either $\rho = 1$ and the whole spectrum of Γ is on the unit circle $|\lambda| = 1$, or else $\rho > 1$ and the spectrum of Γ consists of the simple points ρ and $1/\rho$, and of points on the unit circle. The structures of Lorentz mappings in case $\rho > 1$ (hyperbolic), and in case $\rho = 1$ but Γ has a fixed u interior to K (elliptic), are described. For example, in the elliptic case, H_u and the line through u reduce Γ, and Γ is a unitary transformation of H_u onto itself under the norm $[y, y]^{\frac{1}{2}}$. The remaining (parabolic) case is left to a later study by means of the motions (isometries) in the infinite dimensional Lobachevskyan space defined by remetrizing the set of all x such that $(Jx, x) = 1$ by $d(x, y) = \cosh^{-1} (Jx, y)$.

Section 6 begins the study of completely continuous operators which takes up the remainder of the paper: hereafter every cone K spans B (i.e., $K + (-K)$ is dense in B). Theorem 6.1. If the completely continuous linear operator A takes K into K and (a) has a point other than 0 in its spectrum, there exist a positive characteristic number ρ of A, not less in modulus than all other characteristic numbers of A, and vectors v in K and φ in K^* with $Av = \rho v$, and $A^* \varphi = \rho \varphi$. Theorem 6.2. If (a) of 6.1 is replaced by (a') there is a u in K with $\|u\| = 1$, a $c > 0$, and an integer p such that $A^p u \geqq cu$, 6.1 holds with $\rho \geqq c^{1/p}$. Define A strongly positive to mean that for each x in K there is an integer n with $A^n x \gg 0$. Theorem 6.3. Let K be a cone with interior and let A be a completely continuous operator with $AK \subseteq K$. Then A is strongly positive if and only if (i) A has interior to K precisely one characteristic vector v, $Av = \rho v$, $\rho > 0$; (ii) the conjugate operator A^* has in K^* precisely one characteristic vector φ; this φ corresponds to the same ρ and is strictly positive; and (iii) the corresponding characteristic number ρ is simple and exceeds in modulus all other characteristic numbers of A.

Theorem 6.3 implies that a strongly positive A has a decomposition $Ax = \rho \varphi(x) v + A_1 x$, where $v \gg 0$, φ is strictly positive, $\varphi(v) = 1$, $Av = \rho v$, $A^* \varphi = \rho \varphi$, and the spectrum of A_1 is interior to the circle $|\lambda| = \rho$: that is, $\lim_n \|A_1^n\|^{1/n} < \rho$. Weaker decompositions are obtained in § 7 under weaker hypotheses. Theorem 7.2. If A is completely continuous and B has a new norm μ so that $\mu(Ax) \leqq \mu(x)$ for all x, then either $\lim_n \|A^n\|^{1/n} < 1$, or A has characteristic numbers of modulus 1 and $A = \sum_{i=1}^{k} \epsilon_i P_i + A_1$, where $\lim_n \|A_1^n\|^{1/n} < 1$, ϵ_i are distinct complex numbers of modulus 1, P_i are projec-

Source: Mathematical Reviews, Vol 10 No. 4

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tions of U on finite-dimensional subspaces of B, $P_jP_k=\delta_{jk}P_k$ and $A_1P_j=P_jA_1=0$. Theorem 7.3. If the μ of 7.2 is continuous, then $\mu(P_jx)\leqq\mu(x)$ for all x in H. If $U_n=\sum\epsilon_j{}^nP_j$ also $\mu(U_nx)=\mu(U_1x)\leqq\mu(x)$. These results are applied to an A having a fixed vector v interior to a cone K by showing that $\mu(x)=\inf\{t|-tv\leqq x\leqq tv\}$ is a suitable μ, even continuous if K is normal.

In §8 it is shown how these decompositions can be improved in case K is minihedral. Call U a permutator ($U\bar{H}\subseteq\bar{H}$) if U has a finite-dimensional range and in its range permutes some basis. Then U^* is also a permutator and the characteristic numbers of U are the zeros of $(\xi^a-1)(\xi^b-1)\cdots(\xi^p-1)$, where $a, b, \cdots, p$ are the lengths of the cycles in the permutation associated with U. Say that $g_1, g_2, \cdots, g_n$ lie properly in K if $\sum_{1\leqq i\leqq n}c_jg_j\epsilon K$ if and only if all c_j are nonnegative. Theorem 8.1. Let K be a minihedral cone and $AK\subseteq K$; if in addition A satisfies the conditions of 7.2, then to the conclusions of 7.2 may be added that $U=\sum\epsilon_jP_j$ is a permutator whose basis lies properly in K; hence the characteristic numbers of A of modulus 1 are roots of unity, and the set of fixed vectors of A is nonempty and has a basis lying properly in K. Theorem 8.2. If K is a minihedral closed cone with interior; if A is completely continuous, $AK\subseteq K$ and if A has a fixed vector interior to K, then the conclusions of 8.1 hold and the sets of fixed vectors of A and A^* have biorthogonal bases $v_1, \cdots, v_n$, $\varphi_1, \cdots, \varphi_n$ lying properly in K and K^* such that $\inf(v_i, v_j)=0$ for $i\neq j$. These results are applied to stochastic n-by-n matrices ($a_{ij}\geqq0$ and $\sum_{1\leqq i\leqq n}a_{ij}=1$) and to integral equations in the space of continuous functions with stochastic kernels.

Section 9, related in some of its results to work of Rothe [Amer. J. Math. 66, 245–254 (1944); these Rev. 6, 71] considers certain nonlinear completely continuous monotone operators. For $c>0$, A is called c-monotone with respect to K if it is monotone ($x\leqq y$ implies $Ax\leqq Ay$) and there exists u in K with $|u|=1$ and $Au\geqq cu$. In addition, A is called rectilinear if $Ax=y$ implies $A(\lambda x)=\varphi_x(\lambda)y$, where $\varphi_x(\lambda)$ is real and nonnegative and $\varphi_x(0)=0$. Theorem 9.3. Let A be a completely continuous rectilinear operator c-monotone with respect to K and suppose that for the u appearing in the definition of c-monotony we have $\lim_{\lambda\to0}\varphi_u(\lambda)=\mu>0$. Then A has a characteristic vector (indeed a characteristic ray) in K with positive characteristic numbers.

M. M. Day (Princeton, N. J.).

Source: Mathematical Reviews, Vol 10 No, 4

KREYN, M. G.

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Krein, M. G. On some questions related to the ideas of Lyapunov in the theory of stability. Uspehi Matem. Nauk (N.S.) 3, no. 3(25), 166-169 (1948). (Russian)

The author considers differential equations of the form $dx/dt=F(x,t)$ where x is an element of a Banach space and $F(x,t)$ is an operator defined over the space with values in the space. The relation between the nonlinear equation and its linear approximation is considered, and some theorems are stated which represent generalizations of results due to Perron, Persidsky, and Malkin for the case of systems of ordinary differential equations. No proofs are given.

R. Bellman (Stanford University, Calif.).

Source: Mathematical Reviews, Vol 10 No. 2

KREYN, M.G.

Linear totally continuous operators in functional spaces with two norms. Zbir.prats' Inst.mat.AN URSR no.9:104-129 '48. (MIRA 9:9)
(Operators (Mathematics)) (Functional analysis)

KREYN, M.G.

Krein, M. G. On Hermitian operators with directed functionals. Akad. Nauk Ukrain. RSR. Zbirnik Prac' Inst. Mat. 1948, no. 10, 83–106 (1948). (Ukrainian. Russian summary)

A detailed discussion of and diverse applications for a theorem announced earlier [C. R. (Doklady) Acad. Sci. URSS 53, 3–6 (1946); these Rev. 8, 277]. The main theorem [cf. the review cited] is proved for the case that $\mathfrak{H}$ is an inner product space in which $(f,f)=0$ is possible for $f\neq 0$; one notes also that the matrix-function $T(\lambda)=\|\tau_{jk}(\lambda)\|_{j,k=1}^{p}$ is Hermitian for every λ, that $T(0)=0$, and that $T(\lambda-0)=T(\lambda)$ for every λ. The matrix function which produces the inner product in $\mathfrak{H}$ is unique if and only if at least one of the defect-numbers of the operator A is zero. Of the applications, we here cite the following. Let $\mathfrak{L}$ be the linear space of all p-dimensional vector-functions $g(t)=\{g_1(t),\cdots,g_p(t)\}$ on $[0,a]$ such that $g(0)=g(a)$, and let $A(g)=i\,dg/dt$, for all $g\in\mathfrak{D}(A)=$ the set of g's such that $A(g)$ exists and is in $\mathfrak{L}$. Plainly, the functions $\Phi_j(g,\lambda)=\int_0^a e^{i\lambda t}g_j(t)dt$ satisfy the hypotheses of the main theorem. Let $F(t)=\|f_{jk}(t)\|$, $j,k=1,\cdots,p$, $|t|\leqq a$, be a continuous matrix-function. Writing $(g,f)=\int_0^a\int_0^a\sum_{j,k=1}^{p}f_{jk}(s-t)g_k(s)\overline{f_j(t)}\,ds\,dt$, we see that $\mathfrak{L}$ is an inner product space if and only if

$$\sum_{j,k=1}^{n}(x^{(j)})^{*}F(s_j-s_k)x^{(k)}\geqq 0$$

for all $s_1,\cdots,s_n\in[0,a]$ and $x^{(1)},\cdots,x^{(n)}$ p-dimensional complex vectors. Call the class of all such matrix-functions $\mathfrak{P}_a$. Then the main theorem, applied to this case, asserts that $F(s)=\int_{-\infty}^{\infty}e^{i\lambda s}\,dT(\lambda)$ $(|s|\leqq a)$, for a matrix $T(\lambda)$ as described above, if and only if $F\in\mathfrak{P}_a$.

E. Hewitt.

MATHEMATICAL REVIEWS (Unclassified)
Vol. 14, No. 1, January 1953, pp. 1-120

KREYN, M.G.; KRASNOSEL'SKIY, M.A.; MIL'MAN, D.P.

Defective numbers of linear operators in Banach space and certain geometrical problems. Zbir.prats' Inst.mat.AN URSR no.11:97-112 '48. (MLRA 9:9)
(Operators (Mathematics)) (Topology)

KREYN, M.G.

Principle statements in the theory of representations of Hermitian operators with deficiency index (m, m). Ukr.mat.zhur. [1] no.2:3-66 '49. (MLRA 7:10)

(Operators (Mathematics)) (Spaces, Generalized)

KREYN, M.G.

Hermitian-positive kernels on uniform spaces. Part 1. Ukr.mat.zhur.
[1] no.4:64-98 '49. (MLRA 7:10)
(Topology)

KREYN, M.

Krein, M., and Levin, B. On entire almost periodic functions of exponential type. Doklady Akad. Nauk SSSR (N.S.) 64, 285-287 (1949). (Russian)

An almost periodic function $f(x)$ is said to be of exponential type if it has a largest Fourier exponent Δ and a smallest Fourier exponent $-\Delta$. It will possess an analytic continuation $f(z)=f(x+iy)$ almost periodic in every strip $c<y<d$. Its zeros can be arranged in a sequence $\cdots, a_{-2}, a_{-1}, a_0, a_1, a_2, \cdots$ according to the size of their real parts, and we have then $\Re a_n=(\pi/\Delta)n+\varphi(n)$, where $\varphi(x)$ is almost periodic. We have further $|\Im a_n|\leqq M$, but $\Im a_n$ is not always almost periodic. The function $f(z)$ can be written as a (principal value) product $f(z)=c\prod_{n=-\infty}^{\infty}(1-z/a_n)$ (where $1-z/0$ means z). The author announces the following theorems. If $a_n=(\pi/\Delta)n+\psi(n)$, where $\psi(x)$ is almost periodic with absolutely convergent Fourier series, then the product $\prod_{n=-\infty}^{\infty}(1-z/a_n)$ is an almost periodic function of exponential type with absolutely convergent Fourier series. If on the other hand $f(z)$ is an almost periodic function of exponential type and with absolutely convergent Fourier series and $|a_n-a_{n-1}|\geqq\alpha>0$ for every n, then $a_n=(\Delta/\pi)n+\psi(n)$, where $\psi(x)$ is an almost periodic function with absolutely convergent Fourier series.

H. Tornehave (St. Johns, Que.).

Source: Mathematical Reviews, Vol 10 No. 7

KREYN, M.

Krein, M. Infinite J-matrices and a matrix-moment problem. Doklady Akad. Nauk SSSR (N.S.) 69, 125–128 (1949). (Russian)

A regular J_p-matrix (a generalization of the Jacobi-matrix) is defined as an infinite Hermitian matrix $\{a_{\alpha\beta}\}$ ($\alpha, \beta = 0, 1, \cdots$) having the form $\{A_{ik}\}$ ($i, k = 0, 1, \cdots$), where A_{ik} are p-matrices (i.e., with p rows and p columns), and where A_{ik} is the null-matrix 0 for $|i-k|>1$ and $A_{i,i+1}$ is not singular. A J_p-matrix is related to a matrix-moment problem

$$S_n=\int_{-\infty}^{\infty}\lambda^n dT(\lambda), \qquad n=0, 1, \cdots, \tag{1}$$

where S_n are given p-matrices and $T(\lambda)$ is to be found. The corresponding theory has the features of the classical theory concerning ordinary Jacobi-matrices J_1. Consider the class $\mathfrak{P}$ of all polynomials $P(\lambda)=C_0+C_1\lambda+\cdots+C_n\lambda^n$ ($n\geqq 0$) with the complex variable λ and with C_i belonging to the collection $\mathfrak{M}_p$ of all p-matrices with complex elements. Define $P^*(\lambda)$ by the relation $P^*(\lambda)=[P(\bar{\lambda})]^*$. To a given J_p-matrix A there are attached polynomials $D_k(\lambda)\in\mathfrak{P}$ ($k=0, 1, 2, \cdots$) defined by the recurrent equations

$$A_{k,k-1}D_{k-1}(\lambda)+(A_{k,k}-\lambda I)D_k(\lambda)+A_{k,k+1}D_{k+1}(\lambda)=0$$

($k=0, 1, \cdots$), where $D_{-1}=0$ and D_0 is an arbitrarily chosen nonsingular constant p-matrix [cf., e.g., M. H. Stone, Linear Transformations in Hilbert Space ..., Amer. Math. Soc. Colloquium Publ., v. 15, New York, 1932, p. 546]. The limit $H(z)=\lim_{n\to\infty}(\sum_{k=0}^{n}D_k^*(z)D_k(z))^{-1}$ exists for every complex z, and the rank $r(z)$ of the Hermitian matrix $H(z)$ is the same for all z with $\Im(z)>0$ and analogously for $\Im(z)<0$ [H. Narel, Math. Ann. 112, 247–285 (1936)]. These ranks r_+, r_- are identical with the deficiency indices of the Hermitian operator corresponding to the matrix A. For two polynomials $P(\lambda)=\sum_{k=0}^{m}U_kD_k(\lambda)$, $Q(\lambda)=\sum_{k=0}^{n}V_kD_k(\lambda)$ with constant matrices U_k, V_k the "scalar product" is defined by $\{P, Q\}=\sum_{k=0}^{s}U_kV_k^*$, $s=\min[m, n]$. We have $\{D_i, D_k\}=\delta_{ik}I$, the scalar product is distributive, and $\{CP, Q\}=C\{P, Q\}$, $\{P, CQ\}=\{P, Q\}C^*$. If we put (2) $S_n=\{\lambda^nI, I\}$ ($n=0, 1, \cdots$), we have

$$\sum_{j,k=0}^{n}X_jS_{j+k}X_k^*=\left\{\sum_{j=0}^{n}X_j\lambda^j, \sum_{j=0}^{n}X_j\lambda^j\right\}$$

for any $X_j\in\mathfrak{M}_p$ ($j=0, 1, \cdots$), and we have

Source: Mathematical Reviews, Vol. 11 No. 9

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$$(3)\qquad \sum_{j,k=0}^{n} x_j S_{j+k} x_k^* > 0 \quad (n=0, 1, \cdots)$$

for any vectors $x_j=(\xi_{j1}, \cdots, \xi_{jp})$. Conversely, any sequence $S_n \in \mathfrak{M}_p$ satisfying (3) can be defined by (2) for some properly chosen J_p-matrix A. The condition (3) is necessary and sufficient for the existence of a solution of the matrix-moment problem (1) (with $xT(\lambda)x^*$ nondecreasing in $(-\infty, \infty)$ and with an infinite number of points of increase). This problem has a unique "normed" solution (i.e., $\lim_{\lambda\to-\infty} T(\lambda)=0$, $T(\lambda-0)=T(\lambda)$, $-\infty<\lambda<\infty$) if and only if either r_+ or r_- equals 0. The case of the undetermined matrix-moment problems is considered too, and theorems corresponding to those of the classical theory of the Hamburger moment problem are stated. There are no proofs. The author refers to work in a paper unavailable outside the USSR and to Krasnosel'skiĭ and Kreĭn [Uspehi Matem. Nauk 2, no. 3(19), 60–100 (1947); these Rev. 10, 198]. *O. M. Nikodým.*

Source: Mathematical Reviews, Vol. 11 No. 9

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KREYN, M. G.

PHASE II **TREASURE ISLAND BIBLIOGRAPHICAL REPORT** AID 433 - II

BOOK Call No.: QA263.G3

Authors: GANTMAKHER, F. R. and KREYN, M. G.

Full Title: OSCILLATION MATRICES AND KERNELS, AND SMALL OSCILLATIONS OF MECHANICAL SYSTEMS. 2nd revised edition

Transliterated Title: Ostsillyatsionnyye matritsy i yadra i malyye kolebaniya mekhanicheskikh sistem. 2-e izd.

Publishing Data

Originating Agency: None

Publishing House: State Publishing House of Technical and Theoretical Literature

Date: 1950 No. pp.: 359 No. of copies: 4,000

Editorial Staff: The authors have used the valuable remarks made about the first edition by Doctor V. L. Shmulian of the Academy of Sciences, killed in World War II.

Text Data

Coverage: The book is divided in five chapters and two appendices, which include: a general knowledge of the matrix theory and quadratic forms; theory of oscillation matrices; small oscillations of a mechanical system of n masses in a linear elastic continuum with reference to Jacobian matrices; small oscillations of a mechanical system with an infinite number of degrees of freedom; algebra of matrices with the generalization

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theory of oscillation matrices of "determined sign" with "Markov's vector systems".
Preface: The first edition of this book of 1941 was revised and the changes made in this second edition are mentioned. Chapters II and III have been substantially rewritten. Chapter IV is new. It is a generalization of Chapters II and III, and covers the oscillation of systems with infinite number of degrees of freedom. Its natural mathematical basis is the theory of loaded integral equations with a symmetrical oscillation kernel (nucleus), which the authors state is presented here for the first time in complete form. Chapter V includes generalizations and additions to the algebraic discussions of the preceding chapters, which were dispersed in the text of the first edition. The two appendices are newly added material: 1) on the iterated method of approximate computation of characteristic values of oscillation matrices, and 2) on the application of continued fractions to inverse problems of the oscillation theory. The prerequisite for understanding the book is a thorough knowledge of differential and integral calculus, the theory of determinants and the theory of linear integral equations.
Introduction: Gives the historical development of the study of small oscillations

and their stability (Strum C., Routh, Lyapunov, Hurwitz and especially A. N. Krylov). It mentions the fundamental oscillation properties of a sectional continuum, which are connected with the following property of an oscillation matrix noted by M. G. Kreyn (1934): all minors of such a matrix of any order (principal and secondary) are non-negative. The authors' investigations resulted in deducting an important class of "oscillation matrices" fully non-negative square matrices A, a certain order of which A^k is fully positive. The significance of this is that the fundamental spectral properties of these matrices are in no way connected with the symmetry of the matrix and are common to symmetric and non-symmetric oscillation matrices. The introduction of the new term "oscillation matrices" was made by the authors in connection with the fact that (p. 10) "as soon as a matrix of the influence coefficients of any finite system of masses continuously distributed over a linear elastic structure (such as a rod, or string with beads, secured at both ends) is oscillatory, all the fundamental oscillatory properties of the entire structure are provided for any distribution of masses over it". The total complex of oscillation theorems for boundary value problems (p. 11) of the 2nd and 4th order, as well as for equations of many other types, according to the authors, is deduced as a corollary to the general theory of integral

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equations with an oscillation kernel (nucleus), as far a Green's functions of these differential equations are oscillation kernels. Kellog's theory of his class of integral equations, with the application of Chebychev's functions and Markov's boundary values of integrals, is included.

Abstract: The coverage, preface, introduction and table of contents well cover the subject matter of the book and give complete information on the text. The present abstract includes only the items which need to be elaborated or which were not found in available English texts.

In Chapter II The deduction of the properties of a characteristic of an oscillation matrix (p. 127) is made along the lines of a normal Jacobian matrix. When discussing the characteristic values of an oscillation matrix as functions of its elements, it is demonstrated (p. 133): 1) that the characteristic value λ_j of an oscillation matrix is a non-decreasing function of its elements; 2) that the minimum characteristic value λ_j of a normal Jacobian symmetric matrix (which becomes an oscillation matrix when its elements are positive) is a decreasing function of its elements; and 3) that the maximum characteristic value λ_n of such a matrix is an increasing function. The chapter contains 17 theorems, several lemmas and a number of corollaries.

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Chapter III is devoted mainly to the deliberations and deductions of M. G. Kreyn, co-author of the book, who established that the influence function of a segment continuum, secured at its ends, represents a Kellog's kernel. He established also the importance of Rolle's theorem in the mechanical meaning of the oscillation property of the influence function, According to the authors, they are the first to have done this. They also assert that all the oscillation properties of a continuum proper follow from the oscillation properties of its influence function, which, as mentioned above, is often the kernel of oscillation. The chapter includes eight theorems.

Chapter IV. The oscillation kernel, has been given a new definition in order to construct for the first time (according to the authors) a complete theory of loaded integral equations (pp. 225-242). The Kellog's integral equations theory is expanded to cover the behavior of fundamental functions not only in the limits of the basic integral, but on all the set of its moving points. The results of M. G. Kreyn's investigation follow: 1) a more simple solution of the problem of the fundamental harmonic tone and overtone (pp. 242-248); 2) Kreyn's theorem on the nodes of forced oscillation of a rod with a hinged supported end under the action of a couple with a pulsating momentum applied to the rod at the hinged end (pp. 248-259); 3) the theory of single pair oscillation kernels, developed by Kreyn in 1936 in his paper on integral equations with oscillating fundamental functions, and the formulae for Fredholm's determinant and minor, generalized by the authors for the case of pair kernels (p. 260); 4) a new theorem given and proven (p. 281) on the necessary and sufficient conditions for Green's function $K(x,s)$ $(a \leq x, s \leq b)$ of the Sturm-Liouville regular boundary problem to be an oscillation kernel. The chapter contains 17 theorems.

Chapter V with the exception of the theorems of Feckete, Schoenberg and Markov, the results of the discussions in this chapter belong to the authors. The chapter includes 20 theorems.

Supplement I represents a generalization of L. Collatz's rule.

Supplement II includes M. G. Kreyn's results on the mechanical explanation and adaptation of the work of Stieltjes on continued fractions of a certain type connected with the momentum problem, to the oscillation theory.

Evaluation: A thorough analysis of the text and comparison with available English books show that a comparable book treating exclusively the theory

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of the application of oscillation matrices to small vibrations of mechanical systems, could not be found. The book is mainly based on periodical literature, as shown in the bibliography.

The treatment of the subject and the system adopted in the deductions, statements, discussions and the very short definitions is different from what is generally used in English texts. The book seems to be a very advanced course, making heavy demands upon the mathematical attainments of the reader. No examples from mechanics are given to illustrate the numerous statements and problems, in many instances rather complicated, though abbreviated in their presentation. New terms are introduced without explanations or references. The bibliography is rather voluminous. Some of the chapters include theories newly advanced by the authors. The theories of linear integral equations introduced by Hilbert and of integro-differential equations of Volterra, together with a somewhat original treatment of the matrix theory, form the basis of the text, and are applied to the theory of oscillations.

Purpose: The authors state that the book is written not only for mathematicians, but also for engineers, who are interested in the mathematical theory of oscillations. Its purpose is to introduce the reader into the oscillation matrix theory within a strictly delineated area to which a specific mathematical treatment is given.

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KREYN, M.G.

Krein, M. G. Hermitian-positive kernels in homogeneous spaces. II. Ukrain Nat. Žurnal 2, 10-59 (1950). (Russian)

[Part I, which was in vol. 1 of this journal, is not available.] Let G be a locally compact Abelian group and let $\hat{G}$ be its character group. Let $A(\hat{G})$ denote the Banach algebra of all bounded Radon measures μ on $\hat{G}$ with convolution as multiplication and $\|\mu\|$ total variation of μ. The Fourier-Stieltjes transform maps $A(\hat{G})$ isomorphically onto the algebra R_G of all functions on G which are finite linear combinations of continuous positive definite functions, multiplication in R_G being the usual pointwise multiplication of functions. If f is the correspondent in R_G of μ in $A(\hat{G})$ and $f=f_1-f_2$ where f_1 and f_2 are positive definite then $\|\mu\|=\inf(\|f_1\|+\|f_2\|)=\inf(f_1(e)+f_2(e))$ where the inf is taken over all representations of f as such a difference. Thus an algebra R_G algebraically and topologically isomorphic to $A(\hat{G})$ may be defined without reference to $\hat{G}$ and this definition has meaning when G is neither locally compact nor Abelian. In this paper the author makes the suggested definition and studies the properties of the resulting class of Banach algebras. Actually, somewhat more than half of the paper is concerned with a still more general notion. Let H be a closed subgroup of the topological group G and let Q be the homogeneous space of left H cosets. Let R_{QG} be the linear span of the continuous positive definite functions on $Q\times Q$ which are invariant under the transformations $q, p \rightarrow sq, sp$ $(q, p \in Q, s \in G)$. Then R_{QG} is closed with respect to multiplication and may be normed by analogy with R_G. Here $f(e)$ is replaced by the constant value of $f(p, p)$. When $H=\{e\}$, R_{QG} reduces in an obvious manner to R_G.

The following is a partial list of results. R_{QG} is always a Banach algebra. If Q is discrete then R_{QG} is a conjugate space of a space of functions in such a manner that the positive definite functions correspond to the positive linear functionals. In the definition of the norm the inf concerned is actually achieved. Let Z be the set of all elementary continuous positive definite functions on G. Then $f \in R_G$ if there exists $M>0$ such that $|\sum_{j=1}^{n} c_j f(s_j)| \leq M \sup_{x\in Z} |\sum_{j=1}^{n} c_j x(s_j)|$ for all $s_1, s_2, \cdots, s_n$ in G and all complex numbers $c_1, c_2, \cdots, c_n$.

Source: Mathematical Reviews, Vol 12 No. 9.

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If G is locally compact, this condition is also necessary. If G is compact then R_G consists precisely of all f on G such that $f(x)=\sum_\alpha \mathrm{Tr}\,(A_\alpha U_\alpha(x))$ and $\sum_\alpha \|A_\alpha\|_1<\infty$ where U_α is a generic irreducible representation of G, A_α is a matrix and $A\rightarrow\|A\|_1$ is a certain norm for matrices. If G is compact R_G is analytically complete in the sense that $\varphi(f)$ is in R_G whenever f is in R_G and φ is an analytic function whose domain includes the range of f. An expansion analogous to that above may be written down for functions on Q and the set of all functions on Q for which the expansion is valid may be made into an analytically complete Banach algebra. Let $\mathfrak{A}$ be the set of all continuous functions on Q whose transforms under G span a finite dimensional vector space. If F is a linear functional on $\mathfrak{A}$ such that $F(|k|^2)\geqq 0$ whenever $k\in\mathfrak{A}$ then $F(k)\geqq 0$ whenever k is a nonnegative element of $\mathfrak{A}$. If $\mathfrak{A}$ and $\mathfrak{A}'$ belong to Q and Q' respectively and if $\mathfrak{A}$ and $\mathfrak{A}'$ are isomorphic as *-algebras, then Q and Q' are homeomorphic.

In the next to the last section of the paper the duality between compact groups and quadratic block algebras announced earlier [Doklady Akad. Nauk SSSR (N.S.) 69, 725–728 (1949); these Rev. 11, 491] is worked out in detail. Many of the other ideas in this paper were announced by the author in a series of notes [C. R. (Doklady) Acad. Sci. URSS (N.S.) 28, 13–17, 18–22; 29, 275–280, 355–359 (1940); 30, 5–8, 9–12, 484–488 (1941); these Rev. 2, 315, 316].

G. W. *Mackey* (Cambridge, Mass.).

Source: Mathematical Reviews, Vol 12 No.

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KREYN, M.G.

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Zuhovickiĭ, S. I., and Kreĭn, M. G. Remark on a possible generalization of the theorems of A. Haar and A. N. Kolmogorov. Uspehi Matem. Nauk (N.S.) 5, no. 1(35), 217–220 (1950). (Russian)

Let Q be a compact point set, and $F_1(q), F_2(q), \cdots, F_n(q)$ be real-valued continuous functions on Q. For every function $\Phi(q)$ on Q there is a "polynomial" $\sum c_i F_i(q)$ of best approximation. A. Haar showed that this polynomial is unique for every continuous $\Phi(q)$ if and only if the system $\{F_i(q)\}$ is a Chebyshev system, i.e., if $\sum \gamma_i F_i(q)$ can vanish at $n-1$ points at most, provided $\sum |\gamma_i| \neq 0$ [Math. Ann. 78, 294–311 (1917)]. This result was extended by Kolmogoroff [Uspehi Matem. Nauk (N.S.) 3, no. 1(23), 216–221 (1948); these Rev. 10, 35] to the case of approximation of complex-valued functions by complex-valued polynomials. In the present note the authors generalize the result to vector functions and to hypercomplex functions. *A. Zygmund.*

Source: Mathematical Reviews, Vol 11 No. 9

KREYN, M.G.

Krein, M. G. **On an application of the fixed point principle in the theory of linear transformations of spaces with an indefinite metric.** Uspehi Matem. Nauk (N.S.) 5, no. 2(36), 180–190 (1950). (Russian)

The following theorem is proved. Let

$$J(x, y) = \sum_{j,k=1}^{n} g_{jk}\xi_k\bar{\eta}_j$$

be an Hermitian form which, upon transformation by SU, where U is unitary and S is a diagonal matrix, assumes the form $\sum_{j=1}^{p}\xi_j\bar{\eta}_j - \sum_{j=p+1}^{n}\xi_j\bar{\eta}_j$. Let A be a linear transformation such that $J(Ax, Ax) \geqq J(x, x)$ whenever $J(x, x) \geqq 0$. Then there exists a p-dimensional A-invariant subspace in which $J(x, x)$ is non-negative and in which all characteristic roots of A have absolute value $\geqq 1$. This assertion, under the stronger hypothesis $J(Ax, Ax) = J(x, x)$ for all x, is due to Frobenius [J. Reine Angew. Math. 84, 1–63) (1877)]. This weaker theorem was extended to l_2, $(J(x, x) = \sum_{j=1}^{p}\xi_j\bar{\xi}_j - \sum_{j=p+1}^{\infty}\xi_j\bar{\xi}_j)$ by Pontryagin [Izvestiya Akad. Nauk SSSR. Ser. Mat. 8, 243–280 (1944); these Rev. 6, 273]. (A similar extension is given here for the stronger theorem.) The proof, which is brief, depends upon an ingenious application of Brouwer's fix-point theorem. *E. Hewitt* (Seattle, Wash.).

MATHEMATICAL REVIEWS (Unclassified)
Vol. 14, No. 1, January 1953, pp. 1-20

KREYN, M.G.

Krein, M. G. A generalization of some investigations of A. M. Lyapunov on linear differential equations with periodic coefficients. Doklady Akad. Nauk SSSR (N.S.) 73, 445–448 (1950). (Russian)

The author discusses the stability of solutions of the vector differential equation $dx/dt=\lambda JH(t)x$, λ a parameter, $H(t)$ a $2m$ by $2m$ real symmetric periodic matrix, and J is the matrix $\begin{pmatrix} 0 & I_m \\ -I_m & 0 \end{pmatrix}$, where in turn I_m is the m by m identity matrix. Conditions on λ are obtained, which imply that all solutions of the equation are bounded.

J. G. Wendel (New Haven, Conn.).

Source: Mathematical Reviews, Vol 12, No. 2 .

KREYN, M.G.

Krein, M. G. On the Sturm-Liouville boundary problem in the interval $(0, \infty)$ and on a class of integral equations. Doklady Akad. Nauk SSSR (N.S.) 73, 1125–1128 (1950). (Russian)

Starting from two real functions $\psi(x)$, $\chi(x)$ and a bounded nondecreasing distribution function $\sigma(x)$, the author sets up the integral equation $\varphi(x)=\lambda\int_0^\infty K(x,s)\varphi(s)\,d\sigma(s)$, where $K(x,s)=\psi(x)\chi(s)$ for $x\leqq s$, $K(x,s)=\psi(s)\chi(x)$ for $x\geqq s$. He also defines $\psi(x;\lambda)$, $\chi(x;\lambda)$ by

$$\psi(x;\lambda)=\psi(x)+\lambda\int_0^x V(x,s)\psi(s;\lambda)\,d\sigma(s),$$

and similarly for $\chi(x;\lambda)$, where $V(x,s)=\psi(s)\chi(x)-\psi(x)\chi(s)$,

Source: Mathematical Reviews, Vol 12 No. 5

and further

$$D_0(\lambda)=1-\lambda\int_0^\infty \psi(s;\lambda)\chi(s)d\sigma(s),$$

$$D_1(\lambda)=-\lambda\int_0^\infty \psi(s;\lambda)\psi(s)d\sigma(s),$$

$$E_0(\lambda)=\lambda\int_0^\infty \chi(s;\lambda)\chi(s)d\sigma(s),$$

$$E_1(\lambda)=1+\lambda\int_0^\infty \chi(s;\lambda)\psi(s)d\sigma(s),$$

where $D_0(\lambda)$ is the Fredholm determinant of the first integral equation. Theorem 1 then asserts that for real α, the function $F_\alpha(\lambda)=(\cos\alpha E_0(\lambda)+\sin\alpha E_1(\lambda))/(\cos\alpha D_0(\lambda)+\sin\alpha D_1(\lambda))$ maps the upper half λ-plane into a part thereof. From theorems of Čebotarev [Čebotarev and Meĭman, The Routh-Hurwitz problem for polynomials and entire functions . . . , Trudy Mat. Inst. Steklov. 26 (1949), chapters IV, V; these Rev. 11, 509] it is said to follow that the zeros of the denominator and numerator are real, simple, and separate one another. A partial fraction expansion of $F_\alpha(\lambda)$ is also given. Theorem 2 states that $D_0(\lambda)$, $D_1(\lambda)$, $E_0(\lambda)$, and $E_1(\lambda)$ belong to the class (N) of integral functions $f(z)$ for which $\lim_{|z|\to\infty}|z|^{-1}\log|f(z)|<\infty$, and

$$\int_{-\infty}^{\infty}(1+\lambda^2)^{-1}|\log|f(\lambda)||\,d\lambda<\infty.$$

Theorem 3 deals with the zeros of $D_0(\lambda)$, also the eigenvalues of the first integral equation. It is stated that if $\sum_{\lambda_j\neq 0}|\lambda_j|^{-1}<\infty$, then $D_0(\lambda^2)$ belongs to the class (N) and the limit $\lim_{n\to\infty} n^2\lambda_n^{-1}$ exists [see the author, Bull. Acad. Sci. URSS. Sér. Math. [Izvestiya Akad. Nauk SSSR] 11, 309–326 (1947), theorem 5, the proof of which is said to need correction; these Rev. 9, 179]. Theorem 4 gives an application to the Sturm-Liouville problem of

$$\varphi''+q(x)\varphi+\lambda\rho(x)\varphi=0$$

over $(0,\infty)$, in the "degenerate" case in which for every complex λ there are two solutions for which $\int_0^\infty\varphi^2\rho dx<\infty$. If $\int_0^\infty\rho dx=\infty$ then certain boundary problems have negative eigenvalues $\lambda_{-1}, \lambda_{-2}, \cdots$ for which $\sum_{n=1}^\infty|\lambda_{-n}|^{-1}=\infty$.

F. V. Atkinson (Ibadan).

Source: Mathematical Reviews, Vol 12 No. 5

~~KREYN, M.G.~~

KREYN, M.G.

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~~Krein, M. G.~~ On a one-dimensional singular boundary problem of even order in the interval $(0, \infty)$. Doklady Akad. Nauk SSSR (N.S.) 74, 9-12 (1950). (Russian)

The author develops for the general $2n$th order self-adjoint differential equation the theory of the resolvent, of transform formulae, and of spectral matrix functions; an alternative approach to the latter has recently been given by Levitan [see the preceding review]. It is suggested that the results are to some extent new even in the second-order case. The paper consists almost entirely of definitions and theorems; no proofs being given.

The boundary problem has the form $D^{(2n)}y - \lambda y = 0$, with initial condition at $x=0$ of the form $\sin A\,\mathfrak{y}_1 + \cos A\,\mathfrak{y}_2 = 0$, where $D^{(2n)}y$

$$= p_0 y - \frac{d}{dx}\left(p_1\frac{dy}{dx} - \frac{d}{dx}\left(p_2\frac{d^2y}{dx^2} - \cdots - \frac{d}{dx}\left(p_n\frac{d^ny}{dx^n}\right)\cdots\right)\right).$$

Here the $p_0, \cdots, p_n$ are defined and measurable for $0 \le x < \infty$, [illegible] and [illegible], $\cdots$, $\int_0^c |p_n| dx$ are finite for $c > 0$, [illegible] is an arbitrary Hermitian matrix, and $\mathfrak{y}_1, \mathfrak{y}_2$ are n-vectors involving the derivatives of y. Further definitions are R_A as one of the resolvents (in a certain sense), $R(x, s; \lambda)$ the corresponding resolvent kernel, $\Phi(x;\lambda)$ and $\Phi^*(x;\lambda)$ as certain n-vectors, or row and column matrices, derived from sets of fundamental solutions of $D^{(2n)}\varphi - \lambda\varphi = 0$, and finally V_A a class of matrix functions $T(\lambda) = \|\tau_{ik}(\lambda)\|_1^n$, with $T(0) = 0$, $T(\lambda - 0) = T(\lambda)$, $-\infty < \lambda < \infty$, and such that the form

$$(T(\lambda)\xi, \xi) = \sum_{i,k=1}^{n} \tau_{ik}(\lambda)\xi_i\bar{\xi}_k$$

is nondecreasing. Theorem 1 then associates with every R_A a unique $T(\lambda) \in V_A$ such that for nonreal z, ζ there holds

$$R(x, s; z) - R(x, s; \zeta) = (z - \zeta)\int_{-\infty}^{\infty} \frac{\Phi^*(x; \lambda)\,dT(\lambda)\,\Phi(s; \lambda)}{(\lambda - z)(\lambda - \zeta)}.$$

If also for some a we have $T(\lambda) = 0$ for $\lambda < a$ then

$$R(x, s; z) = \int_a^{\infty} \frac{\Phi^*(x; \lambda)\,dT(\lambda)\,\Phi(s; \lambda)}{\lambda - z}.$$

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KREIN, M.G.

These results are absolutely and uniformly convergent in every finite square $0 \leqq x, s \leqq L$ $(0 < L < \infty)$. They may also be differentiated up to $n-1$ times with respect to x and similarly for s. Theorem 2 sets up two necessary and sufficient conditions for a matrix function $T(\lambda)$ to be a spectral matrix of a differential operator, the first condition being to the effect that the generalised Parseval equality should hold. This latter determines $T(\lambda)$ uniquely in the case of zero deficiency index. Theorems 3 and 4 give two sets of conditions under which the transform formulae

$$f(x) = \int_{-\infty}^{\infty} F(\lambda)dT(\lambda)\Phi(x;\lambda), \qquad F(\lambda) = \int_0^{\infty} f(s)\Phi^*(s;\lambda)ds,$$

hold together with differentiated forms of these formulae.

F. V. Atkinson (Ibadan).

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Reviews. Vol 17 No. 7

KREYN, M.G.

Kovalenko, K. R., and Krein, M. G. On some investigations of A. M. Lyapunov on differential equations with periodic coefficients. Doklady Akad. Nauk SSSR (N.S.) 75, 495–498 (1950). (Russian)

The authors state without proof a number of results pertaining to the solutions of equations of the type $u''-q(t)u'+\lambda p(t)u=0$, where the coefficients are periodic. These results are related to those of Liapounoff, Krein, Putnam, and Borg [cf. Amer. J. Math. 71, 67–70 (1949); these Rev. 10, 455].

R. Bellman.

Source: Mathematical Reviews, Vol 12 No. 8

KREIN, M. G.

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Mathematical Reviews
Vol 14. No. 10
November 1953
Analysis

6/11/54

Krein, M. G. On the theory of entire matrix functions of exponential type. Ukrain. Mat. Zurnal 3, 164-173 (1951). (Russian)

An entire matrix function of exponential type is a square matrix of order n whose elements are entire functions of exponential type. The author takes the norm $\|A\|$ of the matrix A to be the norm of the associated linear transformation in n-space. His main theorem is a generalization of a theorem he previously proved in the case $n=1$ [Izvestiya Akad Nauk SSSR. Ser. Mat. 11, 309-326 (1947); these Rev. 9, 179]: a matrix function F(z) which admits the representation

$$F^{-1}(z)=C_{-1}z^{-1}+C_0+C_1z+\dots+C_{p-1}z^{p-1}+z^p\sum_{k=1}^{\infty}A_k/(z-a_k),$$

with real a_k and $\sum\|A_k\|/|a_k|<\infty$, is of exponential type and $\int_{-\infty}^{\infty}(1+x^2)^{-1}\log^+\|F(x)\|\,dx<\infty$.

This theorem has the following application to systems of differential equations. Let $\mathcal{J}$ be a nonsingular constant Hermitian matrix, H(t) a Hermitian matrix function whose elements are integrable in (0,w), and consider the system (1) $dx/dt=i\lambda\mathcal{J}H(t)x$, and the matrix equation

$$dU/dt=i\lambda\mathcal{J}H(t)U, \quad U(0)=1.$$

The matrix $U(w,\lambda)$ satisfying (2) is an entire function of λ. If H is nonnegative Hermitian, the author shows that $U(w,\lambda)$ satisfies the conditions of his theorem on entire functions and hence has the properties asserted in that theorem. The case $n=2$ is discussed in greater detail. R. P. Boas, Jr. (Evanston, Ill.).

KREYN, M.G.

Krein, M. G. On the application of an algebraic proposition in the theory of matrices of monodromy. Uspehi Matem. Nauk (N.S.) 6, no. 1(41), 171–177 (1951). (Russian)

Let $\dot{x}=A(t)x$, A a complex periodic matrix (period ω), x a complex n-vector, $x=U(t)x_0$ a general solution ($U(0)=I$); the characteristic roots of $U(\omega)$ are called multipliers. Let $\mathfrak{P}$ be the set of all periodic integrable matrices $H(t)$ such that $(H(t)x, x)>0$ for all x and almost all t; $\bar{\mathfrak{P}}$ is the set of matrices such that $(H(t)x, x)$ is $\geqq 0$, its integral in $(0, \omega)$ being >0. 1) If a non-singular hermitian constant matrix G exists having signature $p-q$, such that $H=GA+A^*G \in \mathfrak{P}$, then p (q) multipliers are numerically greater (smaller) than 1. 2) If $P(t)$ is continuous and Im $(P(t)x, x)<0$ for all t, then n multipliers of $\ddot{x}+P(t)x=0$ are numerically greater and n numerically smaller than 1. 3) If a non-singular hermitian constant matrix G exists having signature $p-q$, such that $GA=-iH$, $H \in \bar{\mathfrak{P}}$, then for real λ all multipliers of $\dot{x}=\lambda Ax$ not lying on the unit circle appear in pairs symmetric with respect to the unit circle, the number of such pairs being $\leqq \inf(p, q)$; if Im $\lambda>0$ (<0), then p (q) multipliers lie inside and q (p) outside the unit circle. 4) If $P \in \bar{\mathfrak{P}}$, all statements of Theorem 3 are true for the system $\ddot{x}+\lambda^2 Px=0$, with $p=q=n$.

J. L. Massera (Montevideo).

MATHEMATICAL REVIEW (unclassified) Vol. [illegible], No 3, pp233-240 March 1953

MATHEMATICAL REVIEW (unclassified) Vol. [illegible], No 3, pp233-240 March 1953

KREYN, M. G.

Krein, M. G. The ideas of P. L. Čebyšev and A. A. Markov in the theory of limiting values of integrals and their further development. Uspehi Matem. Nauk (N.S.) 6, no. 4(44), 3-120 (1951). (Russian)

This is an expository memoir, although the subject matter is given a unified treatment with many new proofs and new generality in some of the results. The problem of the title is to estimate $\int_a^b \Omega(t)d\sigma(t)$, where $\sigma(t)$ is a nondecreasing function, $\Omega(t)$ is continuous, and the first $n+1$ (generalized) moments $\int_a^b u_k(t)d\sigma(t)$ have prescribed values c_k, $\{u_k(t)\}$ being a given sequence of functions with appropriate properties, and $a \leqq \xi < \eta \leqq b$. The original problem had $\Omega(t)=1$, $u_k(t)=t^k$, and arises naturally in the theory of probability. A related problem is the restricted moment problem for $\{u_k\}$: to find necessary and sufficient conditions for a given $\{c_k\}_0^n$ to be the first $n+1$ moments of a nondecreasing function; special cases are the restricted Hausdorff, Stieltjes and Hamburger moment problems, the trigonometric moment problem, the coefficient problem for regular functions with positive real part, and the Pick-Nevanlinna interpolation problem. The first 21 pages give a detailed historical introduction.

The first and longer chapter is devoted to the two general problems just described, and the second chapter to the same problems under the restriction $0 \leqq d\sigma \leqq L\,dt$ with a given L, or more generally $d\phi \leqq d\sigma \leqq d\psi$ with given ϕ, ψ. The methods depending on continued fractions, used in the original form of the problems, are not available in the general case; the author devotes some attention to geometrical methods but his chief reliance is on what he calls the method of maximal mass [see below].

We are given $n+1$ continuous functions $u_k(t)$, and we call a sequence $\{c_k\}_0^n$ positive if $P(t)=\sum_0^n a_k u_k(t) \geqq 0$ implies $\sum_0^n a_k c_k \geqq 0$. The first section deals with the fundamental (known) theorem that positive sequences coincide with the moments of nondecreasing functions $\sigma(t)$; illustrations are given in terms of the special theories mentioned above. The next section deals with maximal mass: for given $\{c_k\}$ consider the set V of all $\sigma(t)$ with the c_k as moments, put $\rho(\xi)=\inf \sum a_k c_k / P(\xi)$ (over all positive $P(t)$; then $\rho(\xi)$ is the maximum over V of $\sigma(\xi+)-\sigma(\xi-)$, the "maximal mass." Section 3 is a general discussion of Chebyshev systems of functions ($P(t)$ has at most n roots if not identically zero); henceforward $\{u_k\}$ is assumed to be such a system. Sections 4 and 5 give a detailed investigation of solutions $\sigma(t)$ of the moment problem which have only a finite number of points of increase. The index is twice the number of points of increase of $\sigma(t)$, one at an endpoint counting only half a time; a solution is called canonical if of index at most $n+2$,

Source: Mathematical Reviews, Vol 13 No 5 and principal if of index $n+1$. Theorems are given on the uniqueness of solutions with maximal mass at an assigned point, and on the existence and uniqueness of principal solutions. Section 6 deals with the variation of the points of increase of $\sigma(t)$ when the masses associated with them are varied. Section 7 deals with the formula of numerical integration associated with a canonical solution of the moment problem, and the solution of the original problem of estimating an integral when $\xi=a$, $\eta=b$. Section 8 solves the general problem of estimating $\int_a^b\Omega(t)d\sigma(t)$ with $n+1$ given moments, following P. G. Rehtman [Zapiski Harkov. Mat. Obščestva (4) 15, 69–80 (1938)]. Sections 9 and 10 deal with the modifications necessary respectively for periodic $u_k(t)$ and for an infinite interval (a, b).

The second part of the paper [pp. 96 ff.] deals with the original problem under the restriction $d\phi \leqq d\sigma \leqq d\psi$, and with the following extremal problem (containing many special problems which have been treated by various authors): given $\{c_k\}_0^n$ and $\lambda(t)$, $\mu(t)$, continuous, with $d\mu>0$, $|d\lambda|<d\mu$, to minimize

$$\int_\xi^\eta |P(t)|\,d\mu(t)+\int_a^b P(t)\,d\lambda(t),\quad P(t)=\sum_0^n c_k u_k(t),$$

under the condition $\sum c_k a_k = 1$.

R. P. Boas, Jr.

Source: Mathematical Reviews. Vol 13 No. 5

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KREYN, M. G.

Krein, M. G. On certain problems on the maximum and minimum of characteristic values and on the Lyapunov zones of stability. Akad. Nauk SSSR. Prikl. Mat. Meh. 15, 323–348 (1951). (Russian)

In the first part of the paper are determined precise upper and lower bounds for the squares of the vibrational frequencies of a string of prescribed length l and total mass M, raised to unit tension, whose density-function $\rho(x)$ $(0 \leqq x \leqq l)$ is subject to $0 \leqq \rho(x) \leqq H$, for given $H > M/l$. The corresponding density-functions are also found; the lowest frequency for example is minimised by concentrating the mass M towards the middle of the string, and maximised by concentrating it towards the ends. Results for higher frequencies are deduced from those for the first by the device of supposing the string pinned down at the nodes. Making $H \to \infty$ the author's results include one due to I. M. Rapoport [Doklady Akad. Nauk SSSR (N.S.) 73, 889–890 (1950); these Rev. 12, 183]. Next various generalisations are summarised. Full results are stated, without proof, for the case in which $\rho(x)$ has a positive lower bound as well as an upper bound. The more general differential equation $y'' - q(x)y + \lambda\rho(x)y = 0$ is noted, and appears to involve no new feature. Corresponding problems for vibrating membranes are stated to be more difficult; however the case of a circular membrane is solved in part. In the next section the foregoing results are applied to deduce approximate formulae for the first characteristic value of a non-uniform string in terms of its moment of inertia about the mid-point. Finally the author passes to Liapounoff zones of stability, i.e. ranges of values of λ such that the solutions of $y'' + \lambda p(x)y = 0$ are bounded on the whole real axis, where $p(x)$ is periodic of period l. One of his results is that the first stability zone includes the range

$$0 < \lambda < (4/l)\left(\int_0^l p_+(x)dx\right)^{-1},$$

where $p_+(x) = p(x)$ $(p(x) \geqq 0)$, $= 0$ $(p(x) \leqq 0)$. This improves a result of G. Borg [Amer. J. Math. 71, 67–70 (1949); these Rev. 10, 456]. There are several other results for this and subsequent stability zones.

F. V. Atkinson.

Source: Mathematical Reviews, Vol 13 No. 4

KREYN, M.G.

Krein, M. G. Solution of the inverse Sturm-Liouville problem. Doklady Akad. Nauk SSSR (N.S.) 76, 21–24 (1951). (Russian)

Denote by $S_q(\alpha, \beta)$ the spectrum of $y''+y(\lambda-q(x))=0$ with boundary conditions

$$\cos\alpha y(0)+\sin\alpha y'(0)=0, \qquad \cos\beta y(l)+\sin\beta y'(l)=0.$$

The author studies "problem A": Given two spectra $S=\{\lambda_n\}$, $S'=\{\lambda_n'\}$, to ascertain the existence of and to determine α, α', β, and $q(x)$ such that $S_q(\alpha, \beta)=S$, $S_q(\alpha', \beta)=S'$. Here the spectra S, S' are to interlace and to satisfy the asymptotic law $\lambda_n, \lambda_n' \sim \pi^2 n^2/l^2$. Theorems of uniqueness for similar problems have been given by G. Borg [Acta Math. 78, 1–96 (1946); these Rev. 7, 382], N. Levinson [Mat. Tidsskr. B. 1949, 25–30 (1949); these Rev. 11, 248], and L. A. Čudov [Mat. Sbornik N.S. 25(67), 451–456 (1949); these Rev. 11, 248]. The solution rests upon the following "idea." "Just as to every Jacobian matrix J corresponds a certain power-moment problem, for each solution of which (i.e. the distribution-function) the matrix J is itself completely determined, so to a second-order differential operator L with boundary condition on one end corresponds a 'generalised' moment problem, for each distribution-function of which is completely determined, together with the boundary condition, the operator itself . . .". There results a procedure whereby problem A may be solved, certain necessary conditions being cited. Finally there are some remarks on corresponding problems for $y''+\lambda\rho(x)y=0$.

F. V. Atkinson (Ibadan).

urce: Mathematical Reviews, Vol 12 No. 8

KREYN, M G.

Krein, M. G. **Determination of the density of a nonhomogeneous symmetric cord by its frequency spectrum.** Doklady Akad. Nauk SSSR (N.S.) 76, 345–348 (1951). (Russian)

The integral equation $\phi(x)=\lambda\int_{-1}^{1}K(x,s)\phi(s)d\sigma(s)$ is considered where $K(s,x)=K(x,s)=\frac{1}{2}(1+x)(1-s)$ for $x\leqq s$. Here $\sigma(-1)=0$ and $\sigma(x)$ is monotone nondecreasing. The eigenvalues are denoted by $\lambda_k=\rho_k^2$. The first result is $\lim n/\rho_n=\pi^{-1}\int_{-1}^{1}[\sigma'(x)]^{\frac{1}{2}}dx$. In the symmetric case

$$\sigma(1)-\sigma(x+0)=\sigma(-x-0)-\sigma(-1)$$

the theorem is given that a necessary and sufficient condition that there exist a symmetric $\sigma(x)$ corresponding to a given $0<\rho_1<\rho_2\cdots$ is the absolute convergence of $\Delta(\lambda)=\prod(1-\lambda/\lambda_n)$, $\lambda_n=\rho_n^2$, and the inequality

$$-\sum 1/(\lambda_{2n}\Delta'(\lambda_{2n}))<\infty.$$

Other results are given. *N. Levinson.*

Source: Mathematical Reviews, Vol 13 No. [illegible]

KREYN, M.G.

USSR/Mathematics - Sturm-Liouville Problem Jul/Aug 52

"The Indeterminate Case of the Sturm-Liouville Boundary Value Problem in the Interval (0,oo)," M. G. Kreyn

"Iz Ak Nauk SSSR, Ser Matemat" Vol XVI, No 4, pp 293-324

Studies the spectra of boundary-value problems generated by the eq $y'' + q(x)y + Lp(x)y = 0$ and establishes the characteristics of the spectrum for the indeterminate case. Cites N. Levinson, "Gap and Density Theorems," New York, 1940; Titchmarsh, "Eigenfunction Expansions," Oxford, 1946; Shohat and Tamarkin, "The Problem of Moments," New York, 1943. Cites 1950 works of Soviet mathematicians B. M. Levitan, N. G.

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Chebotarev, N. N. Meyman, F. R. Gantmakher, I. I. Privalov, on allied subjects of Roth-Hurwitz prolem, oscillational matrices and kernels, density, entire functions, eigen-expansions, etc. Submitted by Acad M. V. Keldysh.

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KREYN, M. G.

Mathematical Reviews
Vol. 14 No. 7
July - August, 1953
Analysis.

Krein, M. G. On some new problems of the theory of oscillations of Sturmian systems. Akad. Nauk SSSR. Prikl. Mat. Meh. 16, 555-568 (1952). (Russian)

The author now [cf. preceding review] solves in detail the inverse problem for the light string bearing particles of masses $m_1, \cdots, m_n$, the solubility conditions being automatically satisfied in the case of a finite number of degrees of freedom. For the string with one end fixed and given spectrum $\lambda_1<\lambda_2<\cdots<\lambda_n$, the minimum total mass is $M_m=[\sum_1^n\lambda_j^{-1/2}|P'(\lambda_j)|^{-1}]^2$ where $P(\lambda)=\prod_1^n(1-\lambda/\lambda_j)$, which is uniquely attained by a system with a certain specified "coefficient of dynamical pliability". Analogous results are given for the cases of both ends fixed, both ends free to move transversely. The solution makes use of continued fractions. The author concludes with a rather general discussion of the case of an infinite number of particles.

F. V. Atkinson (Ibadan).

KREYN, M.G.

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Krein, M. G., and Krasnosel'skii, M. A. Stability of the index of an unbounded operator. Mat. Sbornik N.S. 30(72), 219–224 (1952). (Russian)

Let A be a linear operator from one Banach space E_1 into another E_2 which is closed, the set of whose values $\mathfrak{R}(A)$ is a closed subspace, the set of whose zeros $\mathfrak{L}(A)$ has finite dimensionality $\alpha(A)$, and such that the factor-space $E_2/\mathfrak{R}(A)$ has finite dimensionality $\beta(A)$. The authors prove for the index $\nu(A)=\alpha(A)-\beta(A)$ two results in extension of results of the reviewer [Mat. Sbornik N.S. 28(70), 3–14 (1951); these Rev. 13, 46, where the notation $\kappa(A)$ was used tor $\nu(A)$]. The first is that there is a positive ρ such that for bounded B with $\|B\|<\rho$, $A+B$ satisfies the same conditions as A, with $\nu(A+B)=\nu(A)$, $\alpha(A+B)\leqq\alpha(A)$; the novelties here are that A need not be bounded, E_1 need not be the same as E_2, and the result for $\alpha(A)$. The second result states that $\nu(A+B)=\nu(A)$ if B is bounded and finite-dimensional, or again if B is completely continuous and E_2 has a basis; in this case the extension is partial in that the reviewer did not require the existence of a basis. The authors speculate that the basis requirement is not essential. Full proofs are given, on different lines to those of the reviewer. There is close connection with the authors' joint work with D. P. Mil'man [Sbornik Trudov Inst. Mat. Akad. Nauk Ukrain. SSR 11, 97–112 (1948), not accessible to the reviewer].

F. V. Atkinson (Ibadan).

Source: Mathematical Reviews, Vol 13 No. 9

KREYN, M.G.

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Ahiezer, N. I.; and Krein, M. G. **On a generalization of the lemmas of Schwarz and Loewner.** Har'kov. Gos. Univ. Uč. Zap. 40=Zap. Mat. Otd. Fiz.-Mat. Fak. i Har'kov. Mat. Obšč. (4) 23 (1952), 95–101 (1954). (Russian)

Let $w=F(z)$ $(z=x+iy,\ w=u+iv)$ be any analytic function mapping the half-plane $y>0$ into (a subset of) $v>0$ and taking real values on the part $(-\infty, a)$ of the boundary (class $S(a,\infty)$). Further let $F(i)=i$. The authors describe a circular segment which must contain the values $F(\zeta)$, where ζ is a point of the upper half-plane. Schwarz's lemma and Loewner's lemma [Nevanlinna, Eindeutige analytische Funktionen, Springer, Berlin, 1936, p. 51] are derived as special cases. This result is the special case $n=1$ of the following theorem: a necessary and sufficient condition for the existence of an element F of $S(a,\infty)$ such that $F(z_\alpha)=w_\alpha$, $\alpha=1, 2, \cdots, n$; $F(z_{n+1})=F(i)=iT=w_{n+1}$ $(T>0)$ is that the forms

$$\sum_{\alpha,\beta=1}^{n+1} \frac{w_\alpha-\bar{w}_\beta}{z_\alpha-\bar{z}_\beta}\xi_\alpha\xi_\beta,\quad \sum_{\alpha,\beta=1}^{n} \frac{\Omega_\alpha-\bar{\Omega}_\beta}{z_\alpha-\bar{z}_\beta}\xi_\alpha\xi_\beta$$

are nonnegative, where $\Omega_\alpha=(w_\alpha-Tz_\alpha)(z_\alpha-a)/(1+z_\alpha^2)$. As a consequence the authors deduce that $F(z)$ belongs to $S(a,\infty)$ if and only if both $F(z)$ and

$$\{F(z)-\Re F(i)\}\frac{z-a}{1+z^2}+\frac{\Im F(i)}{1+z^2}(1+a^2)$$

map $y>0$ into $v>0$. *R. P. Boas, Jr.* (Evanston, Ill.).

КРЕЙН, М. Г.

Mathematical Reviews
Vol. 14 No. 7
July - August, 1953
Analysis.

Krein, M. G. On inverse problems for a nonhomogeneous cord. Doklady Akad. Nauk SSSR (N.S.) 82, 669-672 (1952). (Russian)

The author presents a rather concentrated summary of his further progress in the theory of a cord stretched to unit tension between $x=0$ and $x=1$, particularly the determination of the mass distribution $d\sigma(x)$ from the characteristic frequencies and other dynamical properties [see M. G. Krein, same Doklady (N.S.) 76, 345-348 (1951); these Rev. 13, 43]. Let $\lambda_0<\lambda_1<\cdots$ be the characteristic numbers when the cord (then referred to as S_2) is fixed at both ends, $\mu_0<\mu_1<\cdots$ the same for the cord (S_1) fixed at the right end only, the left end being free to move transversely. The first problem is the determination of $\sigma(x)$ from the λ_n. It is necessary and sufficient that $\lim n/\lambda_n^{1/2}$ exist and be finite, and that $\sum_0^\infty \lambda_n^{-2}|D'(\lambda_n)|^{-1}<\infty$, where $D(\lambda)=\prod_0^\infty(1-\lambda/\lambda_n)$; then among all such S_2 with the given spectrum there is a unique symmetrical one, which is also that of least total mass. This confirms a previous conjecture [loc. cit.]; theorem 2 of the previous paper gave slightly different conditions. Next studied is the effect of a pulsating force $A\sin\lambda^{1/2}t$ applied to the free end of S_1, leading to the "coefficient of dynamical pliability" $R(\lambda)=D(\lambda)/D_1(\lambda)$, where $D_1(\lambda)=\prod_0^\infty(1-\lambda/\mu_n)$ (cf. $E(\lambda)/D(\lambda)$ of §4 of the cited paper); $\sigma(x)$ is determined by $R(\lambda)$. Regarding the determination of $\sigma(x)$ from the μ_n, it is necessary and sufficient that the limit $T=\pi\lim(n/\mu_n^{1/2})$ exist and be finite, and that $\sum_0^\infty \mu_n^{-3/2}|D_1'(\mu_n)|^{-1}<\infty$. There is then a unique S_1 with minimum mass. The remainder of the paper is concerned with the properties of a "transition function" $\Phi(t)$, i.e., the displacement of the free end of S_1 as a result of unit force applied at the free end for time t. The theorems give conditions under which $\sigma(x)$ is determined by $\Phi(t)$, and conditions for a given $\Phi(t)$ to be a "transition function" for certain types of cord.

F. V. Atkinson (Ibadan).

PA 240T86

USSR/Mathematics - Stieltjes Integral 21 Dec 52

"A Generalization of Stieltjes' Investigations,"
M. G. Kreyn

"DAN SSSR" Vol 87, No 6, pp 881-884

As has been already noted (F. G. Gantmakher and M. G. Kreyn, "Ostsillyatsionnyye Matritsy i Yadra i Malyye Kolebanniya Mekhanicheskikh Sistem" ("Oscillational Matrices and Nuclei and Small Oscillations of Mechanical Systems"), 1950; M. G. Kreyn, Priklad Matemat i Mekhan, 16, No 5, 1952), Stieltjes' investigations on continuous fractions can be interpreted as an investigation of the oscillations of a tense

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thread loaded by an infinite number of beads. Such an interpretation permits one to consider these investigations as a component part of the general theory of oscillations of a string of any length loaded in an arbitrary manner by masses of any total mass. In this article the author expounds the principal positions assumed in this general theory in their purely analytical aspect; the mechanical interpretation of these statements is omitted because of lack of space. A portion of the results obtained can be considered as a further development and generalization of the Weyl theory of the Sturm-Liouville singular problem with semibounded spectrum. Presented by Acad A. N. Kolmogorov 29 Oct 52.

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KREYN, M. G.

KREYN, M. G.

USSR/Mathematics - Operator Index Jul/Aug 53

"Index of Unbounded Operator," I. Ts. Gokhberg.
Soroki, Moldavian SSR

Mat Sbor, Vol 33 (75), No 1, pp 193-198

Considers: a linear operator A that acts from a certain Banach space E_1 into a certain Banach space E_2; the region D(A) that defines A; the region R(A) of A's values. Assumes that the operators A possess the following properties: Ax=0 has a finite number of linearly independent solutions T(A) of measure a(A); the factor-space $E_2/\overline{R(A)}$ has finite measure

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b(A); the form of A is closed (i.e. A is normally solvable). Demonstrates two theorems. Cites the related work of M. G. Kreyn and M. A. Krasnosel'skiy ("Stability of the Index of an Unbounded Operator," Mat Sbor, Vol 30 (72), 1952). Presented 1 Oct 52.

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KREYN, M.G.

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Krein, M. G. On the trace formula in perturbation theory. Mat. Sbornik N.S. 33 (75), 597–626 (1953). (Russian)

In this paper the author presents a rigorous mathematical study of a formula heuristically derived by Livšic [Uspehi Matem. Nauk (N.S.) 7, no. 1(47), 171–180 (1952); these Rev. 14, 185]. He considers a self-adjoint operator H on a Hilbert space, and a self-adjoint perturbing operator T; given a real function $\Phi(\lambda)$ on the real axis, he derives the trace formula

$$S\{\Phi(H+T)-\Phi(H)\}=\int_{-\infty}^{\infty}\xi(\lambda)\,d\Phi(\lambda) \tag{1}$$

for the trace of operator $\Phi(H+T)-\Phi(H)$ under various hypotheses. The function $\xi(\lambda)$ is independent of Φ and can be calculated from the operators H and T. The trace $S\{A\}$ of an operator A is defined for A in the trace class, the set of completely continuous operators A for which the series of eigenvalues, observing multiplicities, converges absolutely. The trace $S\{A\}$ is defined as this sum.

The author gives first a detailed treatment of the function

$$S(z)=S[(H+T-z\,\text{[illegible]})^{-1}-(H-zI)^{-1}]$$

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KREĬN, M.G.

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which corresponds to $\Phi(\lambda)=(\lambda-z)^{-1}$. When T is in the trace class he shows that $(H+T-zI)^{-1}-(H-zI)^{-1}$ belongs to the trace class for all non-real z. He next proves, in three steps, that formula (1) holds in this case. First, the case where T has a one-dimensional range, for some vector φ and real scalar τ: $T\psi=\tau(\psi,\varphi)\varphi$ for all vectors ψ, in this case we have the explicit formula

$$(2)\qquad \xi(\lambda)=\frac{1}{\pi}\lim_{y\to 0}\arg\Delta(\lambda+iy)$$

for the function $\xi(\lambda)$, where $\Delta(z)=1+\tau((H-zI)^{-1}\varphi,\varphi)$. Second, if T has a finite-dimensional range it may be considered as a succession of one-dimensional perturbations and formula (1) carries over. Third, if T belongs to the trace class it can be approximated by finite-dimensional operators T_i and the corresponding ξ_i converge in $L^1(-\infty,\infty)$ to a ξ for which (1) holds. Using this basic result the author establishes (1) for various classes of functions $\Phi(\lambda)$, although he cannot characterize the set of admissible functions.

CONT

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Крейн М.Г.

Next he establishes a generalization of the trace function $S[\Phi(H+T)-\Phi(H)]$ for which (1) holds although the trace cannot exist in the usual sense.

Finally, he discusses another formula of Livšic; taking $\Phi_\mu(\lambda)$ as 1 for $\lambda \leqq \mu$ and zero elsewhere and substituting formally in (1) gives

$$\xi(\mu) = -S[\hat{E}_\mu - E_\mu] \tag{3}$$

where E_μ is the spectral decomposition of H and $\hat{E}$ that of $H+T$. The author observes that (3) holds only in special cases and gives an example where $\hat{E}_\mu - E_\mu$ does not belong to the trace class, so that $S[\hat{E}_\mu - E_\mu]$ is not defined, and calculates $\xi(\mu)$ for this example.

D. C. Kleinecke.